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SOLVABILITY OF CONVOLUTION EQUATIONS IN $\mathscr{K}_p'$, $p > 1$

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Let S be a convolution operator in the space $\mathscr{K}_p'$, $p > 1$, of distributions in R^n growing no faster than $\exp(k|x|^p)$ for some k. A condition on S introduced by I. Cioranescu is proved to be equivalent to $S*\mathscr{K}_p' = \mathscr{K}_p'$.

We denote by $\mathscr{K}_p'$, $p > 1$, the space introduced in [4] and consisting of distributions in R^n which "grow" no faster than $\exp(k|x|^p)$, for some k.

I. Cioranescu [1] characterized distributions with compact support, i.e. in the space $\mathscr{E}'$, having fundamental solutions in $\mathscr{K}_p'$. We recall that a distribution E is a fundamental solution for $S \in \mathscr{E}'$ if

$$S*E = \delta ,$$

where δ is the Dirac measure and * denotes the convolution. Cioranescu proved that, if S is a distribution in $\mathscr{E}'$ and $\hat{S}$ its Fourier transform, the following conditions are equivalent:

(a) There exist positive constants A, N, C such that

$$\sup_{x \in R^n, |x| \leq A[\log(2+|\xi|)]^{1/q}} |\hat{S}(\xi + x)| \geq \frac{C}{(1 + |\xi|)^N}, \ \xi \in R^n ,$$

where $1/p + 1/q = 1$.

(b) S has a fundamental solution in $\mathscr{K}_p'$.

In this paper we study the solvability of convolution equations in $\mathscr{K}_p'$. If $\mathscr{O}_C'(\mathscr{K}_p' : \mathscr{K}_p')$ is the space of convolution operators in $\mathscr{K}_p'$, we ask the question: Under what condition on $S \in \mathscr{O}_C'(\mathscr{K}_p' : \mathscr{K}_p')$ is $S*\mathscr{K}_p' = \mathscr{K}_p'$? The last equation means that the mapping $u \to S*u$ of $\mathscr{K}_p'$ into $\mathscr{K}_p'$ is surjective.

We prove the following theorem which extends the results of Cioranescu mentioned above.

THEOREM. *If S is a distribution in $\mathscr{O}_C'(\mathscr{K}_p' : \mathscr{K}_p')$ then each of the conditions* (a) *and* (b) *is equivalent to each of the following ones*:

(a′) *There exist positive constants A', N', C' such that*

$$\sup_{z \in C^n, |z| \leq A'[\log(2+|\xi|)]^{1/q}} |\hat{S}(\xi + z)| \geq \frac{C'}{(1 + |\xi|)^{N'}} ; \quad \xi \in R^n ,$$

where $1/p + 1/q = 1$.

(c) $S*\mathscr{K}_p' = \mathscr{K}_p'$.