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QUOTIENTS OF COMPLETE MULTIPARTITE GRAPHS

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The principal result of this paper is the determination of every graph that can be covered by a complete multipartite graph, in the usual topological sense of covering spaces.

Various papers of S.R. Alpert, J.L. Gross, and T.W. Tucker have explicitly recognized that knowing what a given graph covers is helpful in constructing surface imbeddings for it, acknowledging that this approach is the underlying secret in the combinatorial current graph method of W. Gustin. Moreover, the fact that every Cayley graph covers a bouquet of circles is of implicit importance in other work on Cayley graph imbeddings, such as that of A.T. White on the genus of a group.

1. Introduction.

The present paper is a sequel to the work of Gross and Tucker [2], whose terminology is adopted here. In addition, the following notations are used.

Let G be a graph. Then $V(G)$ and $E(G)$ denote respectively the set of vertices and the set of edges of G.

A *k-partite graph* is a graph whose vertices can be partitioned into k cells such that no two vertices within the same cell have an edge adjoining them. In general, it is tedious to determine for a given graph G the minimum number k such that G is k-partite, which is, of course, the chromatic number of G. The *complete k-partite graph* $K_{n_1,\cdots,n_k}$ is the (k-partite) graph with a vertex set partition $V_1, \cdots, V_k$ such that $|V_i| = n_i$ for $i = 1, \cdots, k$, and for each $u \in V_i$, $v \in V_j$, there is an edge between u and v if and only if $i \neq j$.

Given two graphs G_1 and G_2, the notation $G_1(+, d)G_2$ means the *d-fold suspension* of G_1 and G_2, which is defined to be the smallest graph which contains both G_1 and G_2 and such that for every pair of vertices (u, v) with $u \in V(G_1)$, $v \in V(G_2)$, there are exactly d-edges between u and v. The 1-fold suspension of G_1 and G_2 is also denoted by $G_1 + G_2$, and is elsewhere called the "join" of G_1 and G_2.

A graph map $p: K \to K'$ is called a *d-fold pseudocovering* if the inverse image of each point and of each (open) line of K' has d components in K and if for every vertex v of K the degree of v equals the degree of its image $p(v)$. In such a case, the graph K' is said to be a *d-fold pseudoquotient* of K.

A graph map $p: K \to K'$ such that K' is connected is called a