PACIFIC JOURNAL OF MATHEMATICS
Vol. 63, No. 2, 1976

MAPPINGS OF POLYHEDRA WITH PRESCRIBED FIXED POINTS AND FIXED POINT INDICES

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The following problem is studied: If points c_k of a polyhedron and integers i_k are given, when does there exist a selfmap within a given homotopy class which has the c_k as its fixed points and the i_k as its fixed point indices? Necessary and sufficient conditions for the existence of such selfmaps are established if the selfmap is a deformation and the polyhedron is of type W, and if the selfmap is arbitrary and the polyhedron is of type S. It is further shown that there always exists a selfmap of an n-sphere ($n \geqq 2$) which has arbitrarily prescribed locations and indices of its fixed points. The proofs are based on Shi Gen-Hua's construction of selfmaps with a minimum number of fixed points.

1. Introduction. It is known that an arbitrarily given closed and nonempty subset of a polyhedron of type W can always be the fixed point set of a suitable selfmap, and even of a deformation [2]. We now ask what happens if not only the locations of the fixed points, but also their indices are prescribed. More precisely, we deal with the following problem:

If the points c_k of a polyhedron and the integers i_k, where $k = 1, 2, \cdots, m$, are given, when does there exist a selfmap within a given homotopy class which has the c_k as its fixed points and the i_k as its fixed point indices?

The problem is an extension of the well-known one concerning the existence of maps with a minimum number of fixed points, whose most general solution to date is due to Shi Gen-Hua [4]. We use Shi's results and methods to a considerable degree.

We first show that the number, location and indices of the fixed points of a deformation of a polyhedron can be arbitrarily prescribed with the only (obvious) condition that the sum of their fixed point indices equals the Euler characteristic of the polyhedron (Theorem 1). In the case of arbitrary selfmaps the—necessary and sufficient—conditions which the fixed point indices must satisfy are naturally more complicated, and express the fact that the number and the indices of the essential fixed point classes of a map are homotopy invariant (Theorem 2). As in Shi's work [4] the assumptions which are made about the polyhedron are more restrictive in the case of