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EMBEDDING METRIC FAMILIES

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The embedding of a metric space in a Banach space plays an important role in metric space theory. In the present paper we consider the problem of embedding a metric family $X \to D$ in a Banach family. We obtain results under various hypotheses: (1) X a metric fiber bundle, (2) X an extended metric family, and (3) X has the coarse topology for a family of local cross-sections.

In §1 the basic definitions are given and a result is proved for metric fiber bundles. In §2 some general conditions are given which suffice for embedding. §3 studies family metrics which are restrictions of continuous pseudometrics. §4 describes the topology of a metric family in terms of a given family of local sections. In §5 a Banach family is associated with a given map and in §6 this is used to embed a locally sectioned family. In §7 an example is described relating to the question of embedding in a product family and also applying the techniques of §6 in a different way.

1. Definitions. In this section various definitions are given and the embedding question is posed. The question is answered in the case of metric fiber bundles.

Suppose $p: E \to D$ a function. Define $E_d = E(d) = p^{-1}(d)$, for $d \in D$, $E_S = E(S) = p^{-1}(S)$ for $S \subset D$, $E \times_D E = \{(e, e') \in E \times E \mid pe = pe'\}$. A continuous function will be called as map.

DEFINITION 1.1. A $\langle$continuous$\rangle$ [pseudo] metric family is a pair $(p: E \to D, m)$ where $p: E \to D$ is a map, $m: E \times_D E \to R$ is an upper semi-continuous $\langle$continuous$\rangle$ function, and $m \mid E(d) \times E(d)$ is [pseudo] metric.

Usually we speak of E as being a metric family rather than (p, m). Recall that a function $u: Z \to R$ from a topological space Z to the real numbers R is called upper semi-continuous provided that $u^{-1}(-\infty, b)$ is open for all $b \in R$. Note that the "metric family" of [2] is called a continuous metric family here.

Suppose $p: E \to D$ a map. A map $s: U \to E$ is called a local section of p if U is open in D and $ps =$ identity (on U). s gives

$$E(U) \xrightarrow[(1,\, sp)]{} E(U) \times_D E(U) \xrightarrow[m]{} R$$

if E is a metric family. So $B(s, r) = \{e \in E \mid m(e, spe) < r\}$ is open in