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LOCAL AND GLOBAL BIFURCATION FROM NORMAL EIGENVALUES

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This paper studies the bifurcation of solutions of nonlinear eigenvalue problems of the form $Lu = \lambda u + H(\lambda, u)$, where L is linear and H is $o(\|u\|)$ on bounded λ intervals. It is shown that isolated normal eigenvalues of L having odd algebraic multiplicity are bifurcation points, and moreover possess branches of solutions which satisfy an alternative theorem. A related situation is studied, and an application explored.

Introduction. In this paper we study the bifurcation of solutions of nonlinear eigenvalue problems. The equations to be studied are of the form

$$Lu = \lambda u + H(\lambda, u) \tag{0.1}$$

where all operators are defined in a real Banach space $\mathscr{B}$. L is assumed to be linear, bounded or unbounded; I, the identity map; and H, compact and $o(\|u\|)$ near $u = 0$. Clearly, $(\lambda, 0)$ is a solution for each real λ, and these are called the trivial solutions of (0.1). Of more interest are the nontrivial solutions, pairs (λ, u) satisfying (0.1) with $u \neq 0$. In particular, one is interested in how solutions of (0.1) are related to solutions of the linear equation

$$Lu = \lambda u\ . \tag{0.2}$$

The study of this led to the following definition.

DEFINITION. A point $(\lambda_0, 0)$ is a bifurcation point for (0.1) if every neighborhood of $(\lambda_0, 0)$ in $\boldsymbol{R} \times \mathscr{B}$ contains a nontrivial solution of (0.1).

Under quite general conditions, it is easy to show that in order for $(\lambda_0, 0)$ to be a bifurcation point of (0.1), it is necessary that λ_0 be in the spectrum of L.

The first general existence theorem for bifurcation points was obtained by Krasnoseljskii [2]. He considered equations of the type

$$u = \lambda Lu + H(\lambda, u) \tag{0.3}$$

where L is linear and compact, I and H being as before. He proved that if λ_0 is a characteristic value of L having odd algebraic multiplicity, then $(\lambda_0, 0)$ is a bifurcation point.

More recently, Rabinowitz [5] studied the same problem as