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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF RETARDED DIFFERENTIAL DIFFERENCE EQUATIONS

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The asymptotic behavior of the solutions of nonautonomous n^{th} order linear retarded differential difference equations is studied in this paper. It is shown that if the coefficients satisfy certain restrictions, then for any real K there exists a finite dimensional subspace $F(K)$ of the solution space having the following property. For any solution x of the equation one has for all $t > 0$ that $x(t) = x_K(t) + x_r(t)$ where x_k belongs to $F(K)$ and $x_r(t) = 0(\exp(-Kt))$ as $t \to \infty$. As in the author's earlier papers, considering the periodic and almost periodic cases, the spaces $F(K)$ are obtained by treating the nonautonomous equation as a perturbation of an n^{th} order autonomous equation.

1. Introduction and notation. We consider perturbations of the autonomous n^{th} order equation

$$L_0(x(t)) = x^{(n)}(t) + \sum_{k=0}^{m}\sum_{l=0}^{l(k)} c_{lk}x^{(l)}(t - \Delta_k) = 0 \tag{1.1}$$

where $0 = \Delta_0 < \Delta_1 < \Delta_m$ and the c_{lk}, for all pairs (l, k) occurring in (1.1), are real numbers. We assume $m \geqq 1$. We also assume that $l(0) < n$ and that $l(k) < n$, $c_{l(k)k} \neq 0$ for $k = 1, \cdots, m$. The perturbed equations will be of the form

$$L_0(x(t)) = D(x(t)) \tag{1.2}$$

where

$$D(x(t)) = \sum_{h=0}^{V}\sum_{g=0}^{g(h)} q_{gh}(t)x^{(g)}(t - \sigma_h)\ . \tag{1.3}$$

Here it is assumed that the q_{gh} belong to $C^{2n}(-\infty, \infty)$ and there exists an $M_1 > 0$ such that

$$|\, q_{gh}^{(j)}(t)\, | \leqq M_1 \tag{1.4}$$

for $t \in (-\infty, \infty)$, $j \leqq 2n$, and $(g, h) \in B$, where B denotes the set of all prirs (g, h) in (1.3) for which $q_{gh}(t) \not\equiv 0$.

In earlier papers, the author has established, in the cases where the coefficients q_{gh} are periodic [6] or almost periodic [7], that for $K > 0$ and sufficiently large there exist finite dimensional solution spaces $F(K)$ of (1.2) possessing the following properties. Any so-