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# THE SPLITTING OF EXTENSIONS OF SL(3, 3) BY THE VECTOR SPACE $F_3^3$

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**We give two proofs that $H^2(\mathrm{SL}\,(3,3), F_3^3) = 0$. This result has appeared in a paper by Sah, [6], but our methods are relatively elementary, i.e., we require only elementary homological algebra and do a group-theoretic analysis of an extension of SL (3, 3) by $F_3^3$ to show that the extension splits. The starting point is to notice that the vector space is a free module for $F_3(\langle x\rangle)$, where $x$ has Jordan canonical form $\begin{pmatrix}1&1&0\\0&1&1\\0&0&1\end{pmatrix}$. We then can exploit the vanishing of $H^i(\langle x\rangle, F_3^3)$ $i=1, 2$.**

For elementary linear algebra, we refer to [2] and for cohomology of groups, we refer to [1], [4], [5] or [6]. Group theoretic notation is standard and follows [3]. Let $V$ be a 3-dimensional $F_3$-vector space and let SL(3, 3) be the associated special linear group. Let $v_1, v_2, v_3$ be a basis for $V$. Define, for $i, j \in \{1, 2, 3\}$, $i \neq j$, and $t \in F_3$, $x_{ij}(t) \in \mathrm{SL}(3, 3)$ by

$$x_{ij}(t)\colon v_k \longmapsto \begin{cases} v_k & k \neq i \\ v_i + tv_j, & k = i\,. \end{cases}$$

Inspection of the Jordan canonical form shows that all $x_{ij}(t)$, $t \neq 0$, are conjugate in $\mathrm{GL}(3,3) = \{\pm I\} \times \mathrm{SL}(3,3)$, hence in SL(3, 3).

Set $G = \mathrm{SL}(3, 3)$. We let

$$(*) \qquad 1 \longrightarrow V \longrightarrow G^* \xrightarrow{\pi} G \longrightarrow 1$$

be an arbitrary extension of $G$ by $V$ with the above action. We will show (*) is split. We use the convention that $u^* \in G^*$ is a representative (arbitrary, unless otherwise specified) for $u \in G$.

The alternate proof of splitting (given later) is much neater than the first version. The methods are quite different, however, and it seems worthwhile to give two proofs.

LEMMA 1. *Let $x = x_{12}(1)x_{23}(1)x_{13}(-1)$. Then $C_G(x) = \langle x, x_{13}(1)\rangle$. If $t \in G$ is an involution which inverts $x$, then $t$ centralizes $x_{13}(1)$.*

*Proof*. The first statement is elementary linear algebra. Namely, $x$ has a cyclic vector in $V$, so that any transformation which commutes with $x$ is a polynomial in $x$. Since $x$ has minimal polynomial of