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PERIODIC JACOBI-PERRON ALGORITHMS AND FUNDAMENTAL UNITS

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In this paper the author states a class of infinitely many real cubic fields for which the Jacobi-Perron algorithm of a properly chosen vector becomes periodic and calculates explicitly a fundamental unit for each field. The main results of this paper are: Let $m = a^6 + 3a^3 + 3$, $\omega = \sqrt[3]{m}$, m cube free $a \in N$; then the Jacobi-Perron algorithm of $a^{(0)} = (\omega, \omega^2)$ is periodic. The length of the primitive preperiod is four and the length of the primitive period is three. A fundamental unit in $Q(\omega)$ is given by $e = a^3 + 1 - a\omega$.

1. Introduction. The Jacobi algorithm **[9]** which was generalized by Perron **[11]** for any dimension $n \geqq 3$ proceeds as follows. Let $a^{(0)}$ be a vector in R_{n-1}; then the sequence $\langle a^{(v)}\rangle$ is called the Jacobi-Perron algorithm, if, for $a^{(v)} = (a_1^{(v)}, \cdots, a_{n-1}^{(v)})$, $(v = 0, 1, \cdots)$

$$(1.1)\quad \begin{aligned} a^{(v+1)} &= \frac{1}{a_1^{(v)} - b_1^{(v)}}(a_2^{(v)} - b_2^{(v)}, \cdots, a_{n-1}^{(v)} - b_{n-1}^{(v)}), \ (b_1^{(v)} \neq a_1^{(v)};\ v = 0, 1, \cdots) \\ b_i^{(v)} &= [a_i^{(v)}], \qquad (i = 1, \cdots, n-1;\ V = 0, 1, \cdots). \end{aligned}$$

For notation see Bernstein's book [7, pp. 11–18].

The Jacobi-Perron algorithm of a vector $Q^{(0)} \in R_{n-1}$ is called periodic, if there exist two rational integers L and M, $L \geqq 0$, $M \geqq 1$, such that

$$(1.2)\qquad a^{(M+V)} = a^{(V)}, \qquad (V = L, L+1, \cdots).$$

If $\min L = l$, $\min M = m$, then the sequence of vectors

$$(1.3)\qquad a^{(0)}, a^{(1)}, \cdots, a^{(L-1)}$$

is called the primitive preperiod of the Jacobi-Perron algorithm, and the sequence of vectors

$$(1.4)\qquad a^{(L)}, a^{(L+1)}, \cdots, a^{(L+M-1)}$$

is called primitive period. The l and m are called respectively the lengths of the primitive preperiod and period. If $l = 0$, the algorithm is said to be purely periodic. By definition, from any periodic Jacobi-