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BONDED QUADRATIC DIVISION ALGEBRAS

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Osborn has shown that any quadratic algebra over a field of characteristic not two can be decomposed into a copy of the field and a skew-commutative algebra with a bilinear form. For any nonassociative algebra G over a field of characteristic not two, Albert and Oehmke have defined an algebra over the same vector space, which is bonded to G by a linear transformation T. In this paper this process is specialized to the class $\mathscr{A}$ of finite dimensional quadratic algebras A over fields of characteristic not two, which define a symmetric, nondegenerate bilinear form, to obtain quadratic algebras $B(A, T)$ bonded to A. In the main results T will be defined as a linear transformation on the skew-commutative algebra V defined by Osborn's decomposition of A. An algebra in $\mathscr{A}$ is called a division algebra if $A \neq 0$ and the equations $ax = b$ and $ya = b$, where $a \neq 0$ and b are elements in A, have unique solutions for x and y in A. Consequently, a finite dimensional algebra $A \neq 0$ is a division algebra if and only if A has no divisors of zero. A basis for V is said to be orthogonal, if it is orthogonal with respect to the above mentioned bilinear form. An algebra in $\mathscr{A}$ is weakly flexible if the ith component of the skew-commutative product of the ith and jth members of each orthogonal basis of V is 0. If $\mathscr{D}(\mathscr{A})$ denotes the class of division algebras in $\mathscr{A}$ and I denotes the identity transformation on V, then the main results are: (1) $A \in \mathscr{D}(\mathscr{A})$, T nonsingular and $B(A, T)$ flexible imply $B(A, T) \in \mathscr{D}(\mathscr{A})$, (2) if $A \in \mathscr{D}(\mathscr{A})$ and A is weakly flexible, then $B(A, T)$ is weakly flexible if and only if $T = \delta I$ for δ a scalar, and (3) if A is a Cayley-Dickson algebra in $\mathscr{D}(\mathscr{A})$, then $B(A, T)$ is a Cayley-Dickson algebra in $\mathscr{D}(\mathscr{A})$ if and only if $T = \pm I$. Finally, a class of nonflexible quadratic division algebras bonded to Cayley-Dickson division algebras will be exhibited.

1. Introduction. A finite dimensional algebra A with identity element 1 over a field F of characteristic not 2 is called a quadratic algebra in case 1, a, and a^2 are linearly dependent over F for all $a \in A$. Following the conventions used by Osborn **[6]** we shall identify the field F with the subalgebra $F1$ and refer to an element in $F1$ as a scalar. Furthermore, if an element $x \in A$ squares to a scalar but x is not a scalar, x is called a vector. If V is the set of all vectors in A, then A is a vector space direct sum of F and V. For x and $y \in A$, let (x, y) denote the scalar component of xy. Clearly (x, y) is a bilinear form from $A \times A$