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SOME CONVERGENCE PROPERTIES OF THE BUBNOV-GALERKIN METHOD

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We generalize the Bubnov-Galerkin method to approximate the resolvent of the m-sectorial operator associated with a densely defined, closed, sectorial form in a Hilbert space. Some special cases of interest are also discussed.

1. Introduction. The Bubnov-Galerkin method [3] was originally devised to approximate the solutions of the equations of the form

$$(z - A)f = g \tag{1}$$

where A is an operator in a Hilbert space, $\mathscr{H}$, g is a vector in $\mathscr{H}$ and z is a complex number. The method proceeds with solving the following set of equations:

$$\sum_{j=1}^{n} \alpha_j(\phi_i \mid (z - A)\phi_j) = (\phi_i \mid g) \qquad i = 1, \cdots, n\,; \tag{2}$$

where $(.\mid.)$ denotes the scalar product in $\mathscr{H}$ and $\{\phi_i\} \subset \mathscr{D}(A)$ is some linearly independent (l.i.) set in $\mathscr{H}$. $\mathscr{D}(\cdot)$ denotes the domain. The questions of interest are the existence and the convergence of the solutions of equation (2). Until recently, the only cases that received a detailed treatment have been when A is compact, bounded or essentially self-adjoint [3, 6]. However, recently the following result was proven by Masson and Thewarapperuma [2]:

R.1. Let A be symmetric, bounded below by b, z be at a nonzero distance from $[b, \infty)$ and $\{\phi_i\}$ be the orthonormal set formed from $\{A^i h\}$ where h is in $\mathscr{D}(A^i)$ for each i. Then $\lim_{n\to\infty} \|\sum_{j=1}^{n} \alpha_j\phi_j - (z - A_F)^{-1}g\| = 0$, where $\|.\|$ denotes the norm in $\mathscr{H}$ and A_F is the Friedrichs extension of A.

Consider the following set of equations:

$$\sum_{j=1}^{n} \alpha_j[z(\phi_i \mid \phi_j) - t(\phi_i, \phi_j)] = (\phi_i \mid g) \qquad i = 1, \cdots, n\,; \tag{3}$$

where t is a densely defined, closable, sectorial, sesquilinear form in $\mathscr{H}$. The sector of t will be denoted by S and since it causes no loss of generality, the vertex will be taken to be one. In the present note we determine the limit of $f_n = \sum_{j=1}^{n} \alpha_j\phi_j$ as n becomes large.