PACIFIC JOURNAL OF MATHEMATICS
Vol. 65, No. 1, 1976

ON CERTAIN g-FIRST COUNTABLE SPACES

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In this paper strongly* o-metrizable spaces are introduced and it is shown that a space is strongly* o-metrizable if and only if it is semistratifiable and o-metrizable (or symmetrizable); g-metrizable spaces are strongly* o-metrizable and hence quotient π-images of metric spaces. As what F. Siwiec did for (second countable, metrizable and first countable) spaces, we introduce g-developable spaces, and it is proved that a Hausdorff space is g-developable if and only if it is symmetrizable by a symmetric under which all convergent sequences are Cauchy.

1. o-metrizable spaces. Let X be a topological space and d be a nonnegative real-valued function defined on $X \times X$ such that $d(x, y) = 0$ if and only if $x = y$. Such a function d is called an *o-metric* [16] for X provided that a subset U of X is open if and only if $d(x, X - U) > 0$ for each $x \in U$. An o-metric d is called a *strong o-metric* [17] if each sphere $S(x; r) = \{y \in X: d(x, y) < r\}$ is a neighborhood of x; a *symmetric* if $d(x, y) = d(y, x)$ for each x and y; a *semimetric* if d is a symmetric such that $x \in \bar{M}$ if and only if $d(x, M) = 0$.

For a space X, let g be a map defined on $N \times X$ to the power-set of X such that $x \in g(n, x)$ and $g(n + 1, x) \subset g(n, x)$ for each n and x; a subset U of X is open if for each $x \in U$ there is an n such that $g(n, x) \subset U$. We call such a map a *CWC-map* (=countable weakly-open covering map). Consider the following conditions on g:

(1) if $x_n \in g(n, x)$ for each n, the sequence $\{x_n\}$ converges to x,

(2) if $x \in g(n, x_n)$ for each n, the sequence $\{x_n\}$ converges to x, and

(3) each $g(n, x)$ is open.

Note that (1) is equivalent to: $\{g(n, x): n \in N\}$ is a local net at x, and (2) is equivalent to: $\{g^*(n, x): n \in N\}$ is a local net at x, where $g^*(n, x)$ is defined by $x \in g^*(n, y)$ if and only if $y \in g(n, x)$.

X is said to be *g-first countable* [1, 20] if X has a CWC-map satisfying (1); *first countable* if X has a CWC-map satisfying (1) and (3). *Semistratifiable* spaces [8] are characterized by spaces having CWC-maps satisfying (2) and (3); *symmetrizable* spaces [4] by spaces having CWC-maps satisfying (1) and (2); *semimetrizable* spaces [11] by spaces having CWC-maps satisfying (1), (2) and (3).

The following proposition may be found in [18], but we will