PACIFIC JOURNAL OF MATHEMATICS
Vol. 65, No. 2, 1976

STANDARD REGULAR SEMIGROUPS

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We give a structure theorem for a class of regular semigroups. Let S be a regular semigroup, let T denote the union of the maximal subgroups of S, and let $E(T)$ denote the set of idempotents of T. Assume T is a semigroup (equivalently, T is a semilattice Y of completely simple semigroups $(T_y: y \in Y)$). If Y has a greatest element and $e, f, g \in E(T)$, $e \geqslant f$, and $e \geqslant g$ imply $fg = gf$, we term S a standard regular gemigroup. The structure of S is given modulo right groups and an inverse semigroup V in which every subgroup is a single element by means of an explict multiplication. We specialize the structure theorem to orthodox, $\mathscr{L}$-unipotent, and inverse semigroups, and to a class of semigroups with Y an ωY-semilattice.

Finally, we show that S is a regular extension of T by V in the sense of Yamada [19].

Let us first state the structure theorem. Let Y be a semilattice with greatest element. Let V be an inverse semigroup with semilattice of idempotents Y such that each subgroup of V consists of a single element. Let $(I, \circ)$ be a standard regular semilattice Y of left zero semigroups $(I_y: y \in Y)$. Let $(J, *)$ be a standard regular semilattice Y of right groups $(J_y: y \in Y)$. Suppose $I_y \cap J_y = \{e_y\}$, a single idempotent element, and $e_y^* e_z = e_y \circ e_z = e_{yz}$ for all $y, z \in Y$. Let H_y denote the maximal subgroup of J_y containing e_y. Let $i \longrightarrow B_i$ be a homomorphism of $(I, \circ)$ into $P(J)$, the semigroup of right translations of $(J, *)$; let $b \longrightarrow \beta_b$ be a mapping of V into $\operatorname{End}(J, *)$, the semigroup of endomorphism of $(J, *)$, and let g be a mapping of $V \times V$ into $H = \bigcup (H_y: y \in Y)$, a semilattice Y of groups $(H_y: y \in Y)$ (with respect to the multiplication $*$ in J) such that 1(a) $jB_i \in H_{yz}$ for $i \in I_y$ and $j \in J_z$, (b) $J_r\beta_b \subseteqq H_{b^{-1}rb}$, (c) $g(c, d) \in H_{(cd)^{-1}cd}$. 2(a) $hB_{e_y} = h\beta_y = h^* e_y$ for $h \in J$ and $y \in Y$. (b) if $j \in H_z$ and $i \in I_z$, $jB_i = j$ (c) $g(y, z) = e_{yz}$ for $y, z \in Y$. 3(a)

$$\beta_c\beta_d = \beta_{cd}C_{g(c,d)}(xC_z = z^{-1*}x*z \text{ for } x, z \in H)$$

(b) $g(a, bc)*g(b, c) = g(ab, c)*(g(a, b)\beta_c)$. Let $(Y, I, J, V, B, \beta, g)$ denote $\{(i, a, j): a \in V, i \in I_{aa^{-1}}, j \in J_{a^{-1}a}\}$ under the multiplication (4)

$$(i, a, j)(w, b, v) = (i \circ e_{(ab)(ab)^{-1}}, ab, g(a, b)*jB_w\beta_b*v) .$$

We show (Theorem 3.14) that $(Y, I, J, V, B, \beta, g)$ is a standard regular semigroup, and, conversely, every standard regular semigroup is isomorphic to some $(Y, I, J, V, B, \beta, g)$.