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A THEOREM ON PRIME DIVISORS OF ZERO AND CHARACTERIZATIONS OF UNMIXED LOCAL DOMAINS

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Two characterizations of when the prime divisors of zero in a complete local ring are all minimal are given. Then a number of results which characterize an unmixed local domain R of altitude two are proved. Among these are when certain over-rings of R are finite R-modules and when R is a subspace of certain Macaulay over-rings. Finally, some characterizations of when an arbitrary local domain is unmixed are given.

1. Introduction. All rings in this article are assumed to be commutative with identity, and the terminology is, in general, the same as that in [6].

In this paper a number of characterizations of when a local domain R is unmixed are proved. The main emphasis is on the case when altitude $R = 2$ (§§3-5), but some characterizations for arbitrary local domains are given in §6. (One reason why characterizations of R being unmixed when altitude $R = 2$ are of value is that if (L, N) is a local domain such that altitude $L > 2$, then certain monadic transformations Q of L are altitude two local domains, and if L is a subspace of such a Q, then L is unmixed if and only if Q is unmixed (4.15). Another reason is that, in answer to a question asked by M. Nagata in 1956 in [5, Problem 1], D. Ferrand and M. Raynaud showed in 1970 in [2, Proposition 3.3] that there exist quasi-unmixed local domains of altitude two which aren't unmixed. This is considered somewhat more fully in §5.) A number of these characterizations in altitude two are due to a more general theorem concerning when the prime divisors of zero in a complete local ring are all minimal, and others have to do with certain Macaulay over-rings of R.

A brief summary of the results in this paper will now be given. Throughout the remainder of this introduction (R, M) is a local domain such that altitude $R = 2$.

The main result in §2 shows that if (L, N) is a complete local ring and altitude $L \geqq 2$, then every prime divisor of zero in L is minimal and has depth > 1 if and only if there exist a regular element $b \in N$ and $n \geqq 1$ such that $(bL)^{(n)} \subseteq bL$, where $(bL)^{(n)}$ is the intersection of the height one primary components of b^nL (2.2).

In §3, using (2.2), six characterizations of when R (altitude two) is unmixed are given in (3.4) and (3.6). Among these are: $(bR)^{(n)} \subseteq$