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# ON GENERALIZED NUMERICAL RANGES

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**The Luecke's class of operators $T$ on a Hilbert space $H$ for which $\|(T - vI)^{-1}\| = 1/d(v, W(T))$, $v \notin \mathrm{CLW}(T)$, where $\mathrm{CLW}(T)$ is the closure of the numerical range $W(T)$ of $T$, has been generalized by using the concept of generalized numerical ranges due to C. S. Lin. Also it has been shown that the notions of generalized Minkowski distance functionals and generalized numerical ranges arise in a natural way for elements of the Calkin algebra.**

**Introduction.** Throughout this note, by an operator, we mean a bounded linear transformation of a Hilbert space $H$ into itself. Let $B(H)$ be the Banach algebra of all operators on $H$ and $K(H)$, the closed two sided ideal of compact operators in $B(H)$. Let $\sigma(T)$, $\mathrm{CLW}(T)$, $r(T)$ and $|W(T)|$ denote respectively the spectrum, the closure of the numerical range $W(T)$, the spectral radius and the numerical radius of an operator $T$. Con $S$ and Bdry $S$ will denote respectively the convex hull and the boundary of a subset $S$ of the complex plane $C$. We write $d(v, S)$ to denote the distance of $v$ from $S$.

Let $\hat{T}$ be the canonical image of $T$ in the (Calkin) quotient algebra $B(H)/K(H)$. For $T$ in $B(H)$, the spectrum $\sigma(\hat{T})$ and the numerical range $W_e(T)$ of $\hat{T}$ will be called the essential spectrum and the essential numerical range of $T$. We write $r_e(T)$ to denote the spectral radius of $\hat{T}$. Salinas [7, Lemma 2.2] has shown that $r_e(T) = \inf\{r(T+K): K \in K(H)\}$.

Let $C_\rho$ be the class of operators with unitary $\rho$-dilation in the sense of B. Sz-Nagy and C. Foias [5]. In [1], Holbrook has defined generalized Minkowski distance functionals $w_\rho(\cdot)$ $(0 \leqq \rho < \infty)$ on $B(H)$ as $w_\rho(T) = \inf\{u: u > 0 \text{ and } u^{-1}T \in C_\rho\}$.

We list in the following theorem some of the properties of $w_\rho(\cdot)$ which we shall need in the sequel:

THEOREM A (Holbrook [1]). *$w_\rho(\cdot)$ has the following properties:*

(1) $w_\rho(T) < \infty$;

(2) $w_\rho(T) > 0$ *unless* $T = 0$, *in fact*, $w_\rho(T) \geqq 1/\rho \|T\|$;

(3) $w_\rho(vT) = |v| w_\rho(T)$ *for* $v \in C$;

(4) *The function* $w_\rho(\cdot)$ *is a norm on* $B(H)$ *whenever* $0 < \rho \leqq 2$;

(5) *For each* $\rho > 0$ *and* $T \in B(H)$, $w_\rho(T^k) \leqq w_\rho(T)^k$, $k = 1, 2, 3, \cdots$;

(6) $w_\rho(T)$ *is a continuous and nonincreasing function of* $\rho$;