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AN ESTIMATE OF THE NIELSEN NUMBER AND AN EXAMPLE CONCERNING THE LEFSCHETZ FIXED POINT THEOREM

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Given a map $f\colon X \to X$ of a compact ANR and any finite connected regular covering $p\colon \tilde{X} \to X$ to which f admits lifts, then one can compute a certain homotopy invariant $N_H(f)$ if the Lefschetz numbers of the lifts and the relation of the lifts to the covering transformations are known. $H = p_{\#}\pi_1(\tilde{X})$. Every map homotopic to f has at least $N_H(f)$ fixed points. If X is a finite polyhedron, then $N_H(f) \leqq N(f)$, the Nielsen number. The smaller invariant is easier to compute by virtue of its smallness, but it is adequate to discern for example homeomorphisms, h, of manifolds in all dimensions with $L(h) = 0$ and $N(h) \geqq 2$.

1. Introduction. It is known that if X is simply-connected and either a compact topological manifold **[2]** or a finite polyhedron satisfying the Shi condition [**1**, p. 139], then the converse of the Lefschetz Fixed Point Theorem is valid, i.e. if the Lefschetz number $L(f)$ of a map $f\colon X \to X$ is zero, then there is a map $g\colon X \to X$ homotopic to f which has no fixed points. This converse remains valid if the condition of simple-connectivity is relaxed to that of Jiang [**1**, p. 141].

Our objective here is to give examples of manifolds M^n in all dimensions which admit self-maps f (homeomorphisms, in fact) with $L(f) = 0$ such that every map homotopic to f has two or more fixed points.

We will use an approach due to G. Hirsch **[3]** which detects essential Nielsen classes using two-fold covers. In the following section we outline a generalization of this procedure.

2. The generalized Hirsch method. Let X be a compact ANR and $p\colon \tilde{X} \to X$ a finite connected regular covering of X. Let $H = p_{\#}\pi_1(\tilde{X})$. For maps $f\colon X \to X$ which admit lifts $\tilde{f}$,

$$\begin{array}{ccc} \tilde{X} & \xrightarrow{\tilde{f}} & \tilde{X} \\ \downarrow & & \downarrow \\ X & \xrightarrow{f} & X \end{array}$$

we will define a number $N_H(f)$ which is no larger than the Nielsen