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OPEN MAPPING THEOREMS FOR PROBABILITY MEASURES ON METRIC SPACES

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Let S and T denote complete separable metric spaces. Let $P(S)$ denote the collection of probability measures on S and equip $P(S)$ with the weak topology. If $\varphi: S \to T$ is continuous and onto, then φ induces a weakly continuous mapping φ^0 of $P(S)$ onto $P(T)$. We show that φ^0 is open in the weak topology if and only if φ is open. However, φ^0 is always open in the norm topology. Let K be a totally disconnected compact metric space and let S^K denote the set of continuous mappings of K into S. Then there exists a natural mapping π of $P(S^K)$ into $P(S)^K$. Blumenthal and Corson have shown that π is onto. We establish that π is an open mapping in the weak topology.

1. Introduction. Let S be a complete separable metric space and let $C(S)$ denote the algebra of bounded continuous real-valued functions on S. Let $M(S)$ denote the collection of Borel measures on S which have finite total variation $\|\mu\|$. Given $f \in C(S)$ and $\mu \in M(s)$, set $\mu(f) = \int f(s)d\mu(s)$. The weak topology on $M(S)$ is the topology on $M(S)$ induced by $C(S)$. Thus, a neighborhood system at μ in $M(S)$ is given by sets of the form

$$N_\epsilon(\mu; f_1, \cdots, f_n) = \{\nu \in M(S): |(\mu - \nu)f_i| < \epsilon \text{ for } i = 1, \cdots, n\}$$
$$\text{where } \epsilon > 0 \text{ and } f_1, \cdots, f_n \in C(S).$$

Let $M^+(S)$ denote the non-negative measures and let $P(S)$ denote the probability measures in $M(S)$.

Our goal is to establish open mapping theorems for some naturally induced mappings between sets of probability measures. Let φ be a continuous map of S onto T where S and T are complete separable metric spaces. Define φ^0: $M(S) \to M(T)$ by

$$\varphi^0\mu(g) = \mu(g \circ \varphi) \text{ for each } g \in C(T).$$

A result of P. A. Meyer [**9**, p. 126] shows that φ^0 maps $P(S)$ onto $P(T)$. We show that φ^0 is open in the weak topology if and only if φ is open.

Let K be a totally disconnected compact metric space and let S^K