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LOCALIZATION IN FULLY BOUNDED NOETHERIAN RINGS

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The paper defines and studies links between the prime ideals of a noncommutative fully bounded noetherian ring, and their role as obstructions to localizability: a localization with properties similar to those of the localization of a commutative ring at a prime ideal, can be constructed if and only if the equivalence class determined by the links is finite. For rings with polynomial identity, the links are described in more detail via an inductive procedure over the PI-degree, and several examples are constructed.

I. Preliminaries. The attempt to localize a noncommutative noetherian ring at its prime ideals leads to the study of *classical* sets of prime ideals (i.e. finite incomparable sets $\{P_1, \cdots, P_n\}$ such that the associated torsion theory has the Ore condition and the Artin Rees property) and in particular of *clans* (i.e., minimal classical sets). It was shown in [16] that a prime ideal belongs to at most one clan, and that the existence of *enough clans* (i.e. each prime ideals belongs to a clan) amounts to localizability at all prime ideals; cf. this paper for more detail.

The very existence of *nontrivial clans* (i.e., clans with more than one member) is evidence of the presence of links between prime ideals which constitute obstructions to localizability. The purpose of this paper is to define and study these links explicitely.

To do so, we restrict attention to FBN-rings (right- and left-fully bounded noetherian rings), where we have these fundamental results of [9] on Krull dimension κ available: Every uniform module is α-smooth for some ordinal α, i.e., all nonzero submodules have the same Krull dimension α. Every finitely generated α-smooth module has an (essentially unique) α-composition series (called basic series in [9]). The α-composition factors, also called α-critical modules, are characterized as the uniform nonsingular R/P-modules, for the various prime ideals P of the ring R with $\kappa(R/P) = \alpha$. Any $R-R$-bimodule which is finitely generated on both sides, has the same Krull dimension on both sides.

The FBN-assumption is natural for other reasons: It makes the Gabriel correspondence between spec R and the collection of indecomposable injective modules one-to-one, via $E(R/P) = E_P^{n_P}$; i.e.,