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GLOBAL ANALYSIS AND PERIODIC SOLUTIONS OF SECOND ORDER SYSTEMS OF NONLINEAR DIFFERENTIAL EQUATIONS

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We establish the existence of global closed connected sets of solutions of nonlinear operator equations with linearized part a polynomial in λ, bifurcating from characteristic values of odd multiplicity. These results are then applied to finding large periodic solutions of systems of second order nonlinear differential equations.

Introduction. Using Leray-Schauder degree theory P. H. Rabinowitz [10] has shown the existence of global continua of solutions bifurcating from characteristic values of odd multiplicity of the linearized part of completely continuous nonlinear operator equations. By purely local bifurcation results, unrelated to those of Rabinowitz, the author [13] has extended M. S. Berger's [1] variational techniques to find small periodic solutions of systems of second order nonlinear differential equations. In this paper we combine the two methods and develop Rabinowitz's global analysis for nonlinear operators whose linearized part is a polynomial in λ and apply these results to the existence of large periodic solutions of second order differential equations of the form

$$u'' + A_1v' + B_1u + F_1(w, w', w'') = 0$$
$$v'' + A_2u' + B_2v + F_2(w, w', w'') = 0$$

where $w = (u, v)$.

1. Global analysis. We consider the existence of continua of solutions of equations of the form

$$x = \lambda Lx + G(\lambda, x) \tag{1.1}$$

where L is a completely continuous linear map of a real Banach space X into itself and $G(\lambda, x)$ is a completely continuous map [12, p. 9] of $\boldsymbol{R} \times X \rightarrow X$ satisfying $G(\lambda, x) = o(\|x\|)$ for x near zero, uniformly on bounded λ intervals. A solution of Eq. (1.1) is a point $(\lambda, x) \in \boldsymbol{R} \times X$ satisfying Eq. (1.1) and will be called trivial if $x = 0$. A $\lambda_0 \in \boldsymbol{R}$ is said to be a bifurcation point if every neighborhood of $(\lambda_0, 0)$ contains nontrivial solutions of Eq. (1.1). The closure of the set of nontrivial solutions of (1.1) will be denoted S.

A $\lambda_0 \in \boldsymbol{R}$ will be called a characteristic value of a linear operator L if there exists a nonzero $x_0 \in X$ such that $x_0 = \lambda_0 Lx_0$. The set of