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PARAMETRIZED SURGERY AND ISOTOPY

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The pseudo-isotopy techniques of Cerf-Hatcher-Wagoner are combined with surgery theory to give information about the group of isotopy classes of diffeomorphisms of a smooth manifold. For example, for the n-torus, $n \geqq 6$, this group is determined completely. We also provide a geometric interpretation of the periodicity sequence of [11].

Introduction. Let M be an n-dim ($n \geqq 6$) smooth manifold without boundary[1] and let Diff M be the group of diffeomorphisms of M. Let Aut M be the H-space of simple homotopy equivalences of M to itself, i.e., Aut $M = \{f \in M^M \mid f$ is a simple homotopy equivalence$\}$. We have the following fibration

(1) $$\mathfrak{S}(M) \longrightarrow \operatorname{Diff} M \longrightarrow \operatorname{Aut} M .$$

Then, a point in $\mathfrak{S}(M)$ is represented by a pair (ϕ, ϕ_t) where $\phi \in \operatorname{Diff} M$ and ϕ_t is a path in Aut M connecting ϕ to $\operatorname{Id} \in \operatorname{Aut} M$. Set

(2) $$M_\phi = M \times I/\{(m, 1) \sim (\phi(m), 0)\} ,$$

the mapping torus of ϕ. ϕ_t induces a simple homotopy equivalence

(3) $$F\colon (M_\phi, M \times 1) \longrightarrow (M \times S^1, M \times 1) .$$

Following [10], one can construct a space $\mathscr{S}(M \times (S^1, 1))$ of simple homotopy smoothings of $M \times S^1$ which are standard on $M \times 1$. W. Browder [1] studies the map

(4) $$\tau\colon \mathfrak{S}(M) \longrightarrow \mathscr{S}(M \times (S^1, 1))$$

defined by $\tau((\phi, \phi_t)) = F$. On the other hand, we have the map

(5) $$\eta\colon \mathscr{S}(M \times (S^1, 1)) \longrightarrow G/0^{\Sigma M^+}$$

where $\Sigma M^+ = M \times S^1/M \times 1$. Let us consider the following diagram of fibrations

(6) $$\begin{tikzcd} \mathscr{B}(M) \arrow[r, "="] \arrow[d] & \mathscr{B}(M) \arrow[d] & \\ \mathscr{C}(M) \arrow[r] \arrow[d] & \mathfrak{S}(M) \arrow[r, "\eta\cdot\tau"] \arrow[d, "\tau"] & G/0^{\Sigma M^+} \arrow[d] \\ \mathscr{L}_2(M) \arrow[r] & \mathscr{S}(M \times (S^1, 1)) \arrow[r, "\eta"] & G/0^{\Sigma M^+} \end{tikzcd}$$

[1] Everything works for PL, topological manifold, and manifold with boundary if the boundary is only allowed to move by an isotopy. Using a result of K. Igusa, everything works also for $n = 5$.