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INTEGRAL REPRESENTATION OF TCHEBYCHEFF SYSTEMS

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A representation of Tchebycheff systems in terms of iterated Riemann–Stieltjes integrals, is given.

1. Introduction. A system of real-valued functions $\{u_0, u_1, \cdots, u_n\}$ defined on a totally ordered set is called a Tchebycheff system or T-system (Weak Tchebycheff system or WT-system), provided that for every choice of points $t_0 < t_1 < \cdots < t_n$ of the set,

$$D(u_0, u_1, \cdots, u_n / t_0, t_1, \cdots, t_n) = \det(u_i(t_j);\ i, j = 0, 1, \cdots, n) \tag{1}$$

is strictly positive (nonnegative). A function u is said to be convex with respect to the system $\{u_0, u_1, \cdots, u_n\}$, if $\{u_0, u_1, \cdots, u_n, u\}$ is a WT-system. The set of functions convex with respect to $\{u_0, u_1, \cdots, u_n\}$ is evidently a cone. This cone is referred to as "Generalized Convexity Cone". If $\{u_0, u_1, \cdots, u_i\}$ is a T-system for $i = 0, 1, \cdots, n$, then $\{u_0, u_1, \cdots, u_n\}$ is called a Complete Tchebycheff system or CT-system. Note that no assumptions of continuity have been made in this paragraph.

In 1965 there appeared a paper by M. A. Rutman in which the following proposition is stated (cf. [**4**, Thm. 3]):

THEOREM. *Suppose the system of right-continuous functions $\{1, u_1, u_2, \cdots, u_n\}$ is a CT-system on the open interval (a, b). Then there is a system $\{1, y_1, y_2, \cdots, y_n\}$ admitting of the following two representations on (a, b):*

$$y_i = u_i + \sum_{j=0}^{i-1} a_{i,j} u_j; \qquad i = 1, 2, \cdots, n, \tag{2}$$

and

$$\begin{aligned} y_1(t) &= \int_c^t dp_1(s) \\ y_2(t) &= \int_c^t \int_c^{s_1} dp_2(s_2) dp_1(s_1) \\ &\text{-------------------} \\ y_n(t) &= \int_c^t \int_c^{s_1} \cdots \int_c^{s_{n-1}} dp_n(s_n) dp_{n-1}(s_{n-1}) \cdots dp_1(s_1), \end{aligned} \tag{3}$$

where $c \in (a, b)$ is arbitrary, and the functions p_i are strictly increasing and right-continuous on (a, b).