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A NOTE ON SPECTRAL CONTINUITY AND ON SPECTRAL PROPERTIES OF ESSENTIALLY G_1 OPERATORS

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A bounded operator on a separable Hilbert space is essentially G_1 if the image of T in the Calkin algebra satisfies condition G_1. This paper contains results describing (1) isolated points of the essential spectrum of essentially G_1 operators, and (2) essentially G_1 operators whose essential spectrum lies on a smooth Jordan curve. Finally, the continuity of the essential spectrum, Weyl spectrum, and spectrum is discussed.

Notation and definitions. Throughout this paper H denotes a separable Hilbert space, $\mathscr{B}(H)$ denotes all bounded operators on H, $\mathscr{K}$ denotes all compact operators in $\mathscr{B}(H)$, $\mathscr{B}(H)/\mathscr{K}$ denotes the Calkin algebra, and $\pi: \mathscr{B}(H) \rightarrow \mathscr{B}(H)/\mathscr{K}$ denotes the quotient map. Since $\mathscr{B}(H)/\mathscr{K}$ is a C^*-algebra, there exists a Hilbert space H_0 and an isometric *-isomorphism ν of $\mathscr{B}(H)/\mathscr{K}$ into $\mathscr{B}(H_0)$ [see 2]. The *essential spectrum* of $T \in \mathscr{B}(H)$, denoted by $\sigma_e(T)$, is the spectrum of $\pi(T)$ in the Calkin algebra. T is *essentially* G_1 if $\|(\pi(T) - z)^{-1}\| = 1/d(z, \sigma_e(T))$ for all $z \notin \sigma_e(T)$. T is *essentially hyponormal, essentially normal, or essentially self-adjoint* if $\pi(T^*T - TT^*) \geqq 0$, $\pi(T^*T - TT^*) = 0$, or $\pi(T^* - T) = 0$, respectively. If λ is an eigenvalue of T then λ is a *normal eigenvalue* if $\{x \in H: Tx = \lambda x\} = \{x \in H: T^*x = \lambda^* x\}$. If λ is an approximate eigenvalue of T then λ is a *normal approximate eigenvalue* if $\|(T - \lambda I)x_n\| \rightarrow 0$ if and only if $\|(T - \lambda I)^* x_n\| \rightarrow 0$, where $\|x_n\| = 1$ for all n.

1. Spectral properties of essentially G_1 operators.

Recall that an isolated point λ of the spectrum of an operator $T \in \mathscr{B}(\mathscr{K})$ that satisfies condition G_1 (i.e., $\|(T - zI)^{-1}\| = 1/d(z, \sigma(T))$ for all $z \notin \sigma(T)$) must be a normal eigenvalue. What happens when λ is an isolated point of $\sigma_e(T)$ and T is essentially G_1? We first look at the special case when T is a compact operator ($\sigma_e(T) = \{0\}$). The following is a "folk" theorem whose proof is included for completeness.

THEOREM 1. *If T is a compact operator with* $\ker T = \ker T^* = \{0\}$, *then* 0 *is a normal approximate eigenvalue of* T.

Proof. Since T is compact, 0 is in the approximate point spect-