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MEASURES WITH CONTINUOUS IMAGE LAW

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Let M be a topological space, and X a metric space. Let $P(X)$ denote the collection of probability measures on X. Let $C(M, X)$ denote the set of continuous functions from M to X. Let $P(X)$ have the weak topology, and let $C(M, X)$ have the topology of uniform convergence. For a fixed measure $\mu \in P(C(M, X))$, and a member $t \in M$, define a measure $t\mu$ on X by

$$t\mu(A) = \mu\{f \in C(M, X): f(t) \in A\} .$$

In this paper, we consider the following problem: given a continuous function $T: M \to P(X)$, when is there a measure $\mu \in P(C(M, X))$ such that $T(t) = t\mu$ for all t?

This problem has been introduced and studied by Blumenthal and Corson in [2] and [3].

The main results of this paper are as follows:

1. Let F be a closed subset of a totally disconnected compact metric space M, and let $i_F: F \to M$ be the natural inclusion. Let X be a complete separable metric space. Suppose that T is a continuous function from M to $P(X)$, and μ_F is a measure in $P(C(F, X))$ such that $T(t) = t\mu_F$ for all $t \in F$. Then there is a measure $\mu \in P(C(M, X))$ such that $T(t) = t\mu$ for all $t \in M$ and $i_F\mu = \mu_F$. Consequently, the natural map $\phi: P(C(M, X)) \to C(M, P(X))$, defined as $\phi(\mu)(t) = t\mu(\mu \in P(C(M, X)), t \in M)$ is open, and there is a continuous function $\xi: C(M, P(X)) \to P(C(M, X))$ such that $\xi(T) \in \phi^{-1}(T)$.

2. Let M be a separable metric space, and let S^1 be the unit sphere in R^2. Let T be a continuous function from M into $P(S^1)$ such that $\operatorname{Supp} T(t)$ is a connected subarc of S^1 and of arc length $\leqq 2\pi - k$, for all t and some fixed $0 < k \leqq 2\pi$. Then there is a measure $\mu \in P(C(M, S^1))$ such that $T(t) = t\mu$ for all t.

3. Some generalizations of the results in [2] and [3] will be considered.

1. Introduction. Let Y be a metric space. By a measure on Y, we mean a regular Borel measure on the class of all Borel subsets of Y. Let μ be a measure on Y. We define the support of μ to be the smallest nonempty closed subset $F \subseteqq Y$ such that $\mu(U) > 0$ for every open subset U in Y such that $U \cap F \neq \varnothing$. We will denote this