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ON STARLIKENESS AND CONVEXITY OF CERTAIN ANALYTIC FUNCTIONS

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Let N be the class of normalised regular functions

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k , \quad |z| < 1 .$$

For $0 \leqq \lambda < 1, \gamma \geqq 1$, let $f(z), g(z) \in N$ be such that

$$|f(z)/[\lambda f(z) + (1-\lambda)g(z)] - \gamma| < \gamma , \quad |z| < 1 .$$

We establish the radius of starlikeness of $f(z)$ under the assumption $\mathrm{Re}\{g(z)/z\} > 0$, or $\mathrm{Re}\{g(z)/z\} > 1/2$, or $\mathrm{Re}\{zg'(z)/g(z)\} > \alpha, 0 \leqq \alpha < 1$, or $\mathrm{Re}\{1 + zg''(z)/g'(z)\} > 0$ for $|z| < 1$. The analysis may be extended to the problem of finding the radius of convexity for certain subclasses of N.

1. Introduction and notation.

Let S, S^*, S^c denote the subclasses of N which are univalent, univalent starlike, univalent convex in $|z| < 1$ respectively.

A necessary and sufficient condition for $f(z) \in N$ to be univalent starlike in $|z| < r$ is

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > 0 , \quad |z| < r .$$

A necessary and sufficient condition for $f(z) \in N$ to be univalent convex in $|z| < r$ is

$$\mathrm{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0 , \quad |z| < r .$$

A function $f(z)$ belongs to $S^*(\beta)$, i.e., is starlike of order β, $0 \leqq \beta < 1$, if it satisfies the condition

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \beta , \quad |z| < 1 .$$

A function $f(z)$ belongs to $S^c(\beta)$, i.e., is convex of order β, $0 \leqq \beta < 1$, if it satisfies the condition

$$\mathrm{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \beta , \quad |z| < 1 .$$

Let $\mathscr{P}_\alpha$ denote the class of regular functions of the form

$$p(z) = 1 + \sum_{k=1}^{\infty} c_k z^k , \quad |z| < 1 ,$$