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NORMAL CONGRUENCE SUBGROUPS OF THE HECKE GROUPS $G(2^{(1/2)})$ AND $G(3^{(1/2)})$

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Normal congruence subgroups of the classical modular group have been completely classified by M. Newman and D. McQuillan. In this note we begin the classification of normal congruence subgroups of the Hecke groups $G(2^{(1/2)})$ and $G(3^{(1/2)})$.

-Our main result is that if G is a normal congruence subgroup of $G(m^{(1/2)})$, $m = 2, 3$, containing the principal congruence subgroup $\bar{\Gamma}_m(nm^{(1/2)})$ where $(n, 6) = 1$ and if G contains only even elements, then G is $\Gamma_m(dm^{(1/2)})$, $\bar{\Gamma}_m(dm^{(1/2)})$ where $d \mid n$ or $\Gamma_m(d)$, $\bar{\Gamma}_m(d)$ where $d \mid n$ and $d > 1$. To obtain this result we use facts about the level of a congruence subgroup which are of independent interest.

2. Definitions. The set of 2×2 matrices with integer entries and determinant one is the classical modular group which is denoted by $\Gamma(1)$. The principal congruence subgroups of level n are

$$\bar{\Gamma}(n) = \left\{ \begin{pmatrix} ab \\ cd \end{pmatrix} : a \equiv d \equiv +1 \,(\bmod n), b \equiv c \equiv 0 (\bmod n) \right\}$$

and

$$\Gamma(n) = \left\{ \begin{pmatrix} ab \\ cd \end{pmatrix} : a \equiv d \equiv \pm 1 \,(\bmod n), b \equiv c \equiv 0 (\bmod n) \right\}.$$

The modular group and its subgroups have been studied extensively. In particular, M. Newman **[5]** and D. McQuillan **[4]** have shown that if $\bar{\Gamma}(n) \subset G \subset \Gamma(1)$, $(n, 6) = 1$, then $G = \bar{\Gamma}(d)$ or $\Gamma(d)$ where $d \mid n$. The only additional result that we need is

$$\bar{\Gamma}(n)\bar{\Gamma}(k) = \Gamma(1), \qquad (n, k) = 1, \tag{2.1}$$

which was proved by M. Newman and J. R. Smart in **[6]**.

In **[1]** E. Hecke introduced an infinite class of discrete groups $\hat{G}(\lambda_q)$ of linear fractional transformations preserving the upper half plane. $\hat{G}(\lambda_q)$ is the group generated by $Sz = z + \lambda_q$ and $Tz = -1 | z$ where $\lambda_q = 2\cos(\pi | q)$, q an integer, $q \geqq 3$. We are interested in the corresponding matrix groups. When $q = 3$, we have the modular