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C^*-ALGEBRAS OF TRANSFORMATION GROUPS WITH SMOOTH ORBIT SPACE

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We investigate the structure of certain locally compact Hausdorff transformation groups (G, X) and the C^*-algebras $C^*(G, X)$ associated to them. When G and X are second countable and the action is free, we obtain a simple necessary and sufficient condition for $C^*(G, X)$ to be a continuous trace algebra, and show that the continuous trace algebras so arising are never "twisted" over their spectra. When G is separable compactly generated Abelian and X contains a totally disconnected set of fixed points whose complement, Z, is a trivial G-principal fiber bundle over its orbit space Z/G, with Z/G compact, $C^*(G, X)$ can be described completely using the Brown-Douglas-Fillmore theory of extensions of the compact operators on a separable Hilbert space by a commutative algebra. These results yield as special cases the structure of the C^*-algebras for several infinite families of solvable locally compact groups.

In greater detail the contents of the paper are as follows: In § 1 we discuss, after some preliminary lemmas, $C^*(G, X)$ for X having a dense free orbit with complement a finite set of fixed points. (The possible structure of such transformation groups is closely related to the "end theory" of G.) Complete results are obtained when G is separable compactly generated abelian. As an application we compute the group C^*-algebras of the "$ax+b$" groups over all nondiscrete locally compact fields. (The real case was first treated by Z'ep [30], the complex case by Rosenberg [22], while the p-adic result was found independently by Rosenberg and myself. The methods here are considerably less computational than those of [22] and [30], and in particular we avoid solving any integral equations.)

In § 2 we prove a result which permits us to reduce the study of $C^*(G, X)$ in certain cases to the study of algebras of simpler transformation groups. We then apply this to (G, X) for which (1) G is compactly generated, (2) the set Y of fixed points of X is totally disconnected, (3) the complement $Z = X\backslash Y$ of Y is a trivial G-principal fiber bundle [24] over Z/G, and (4) Z/G is compact, obtaining a complete description of $C^*(G, X)$ when G is in addition separable abelian. These results are applied to find the C^*-algebras of some further locally compact groups, including a family of solvable Lie groups whose C^*-algebras were first computed in [22].