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ON COMMON FIXED POINTS FOR SEVERAL CONTINUOUS AFFINE MAPPINGS

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It is known from Markov-Kakutani theorem that if T_j $(j = 1, 2, \cdots, J)$ are continuous affine commuting self-mappings on a compact convex subset of a locally convex space, then the intersection of the sets of fixed points of T_j $(j = 1, 2, \cdots, J)$ is nonempty. The object of this paper is to show a result which says more than the above theorem does, and actually our theorem shows in the case of $J = 2$ that the set of fixed points of $\lambda T_1 + (1 - \lambda)T_2$ always coincides, for each λ $(0 < \lambda < 1)$, with the intersection of the sets of fixed points of T_1 and T_2.

1. Introduction. In this paper, we deal with a commuting family of continuous affine self-mappings on a compact convex subset of a locally convex space, and we give a result which seems to say more than Markov-Kakutani theorem itself does.

Let $F(T)$ denote the set of fixed points of a mapping T.

We have a following main theorem.

THEOREM. *Let K be a compact convex subset of locally convex space X, and let T_j $(j = 1, 2, \cdots, J)$ be continuous affine commuting self-mappings on K. Then $\bigcap_{j=1}^{J} F(T_j)$ is nonempty and equal to $F(\sum_{j=1}^{J} \alpha_j T_j)$ for any α_j $(j = 1, 2, \cdots, J)$ such that $\sum_{j=1}^{J} \alpha_j = 1$, $0 < \alpha_j < 1$ $(j = 1, 2, \cdots, J)$.*

Before proving theorem, we first prove the following lemmas on which the proof of theorem is based.

LEMMA 1. *If T is a continuous affine self-mapping on a compact convex subset K of a locally convex space X, then*

(a) *for any $\varepsilon > 0$, there exists an integer N such that $\varepsilon(K - K) = x_i - Tx_i$ for all x_0 in K and $i \geqq N$, where x_i is defined for each positive integer i,*

$$x_i = (1 - \lambda)x_{i-1} + \lambda T x_{i-1}\ , \quad (0 < \lambda < 1)\ ,$$

(b) *a point of accumulation of $\{x_i\}_{i=0}^{\infty}$ is a fixed point of T.*

Proof. (a) Let I denote an identity mapping on K, then we have