PACIFIC JOURNAL OF MATHEMATICS
Vol. 72, No. 2, 1977

ON THE DISTRIBUTION OF SOME GENERALIZED SQUARE-FULL INTEGERS

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Let a and b be fixed positive integers. Let $n=p_1^{a_1}p_2^{a_2}\cdots p_r^{a_r}$ be the canonical representation of $n>1$ and let $R_{a,b}$ denote the set of all n with the property that each exponent $a_i (1\leqq i\leqq r)$ is either a multiple of a or is contained in the progression $at+b$, $t\geqq 0$. It is clear that $R_{2,3}=L$, the set of square-full integers; that is, the set of all n with property that each prime factor of n divides n to at least the second power. Thus the elements of $R_{a,b}$ may be called generalized square-full integers. This generalization of square-full integers has been given by E. Cohen in 1963, who also established asymptotic formulae for $R_{a,b}(x)$, the enumerative function of the set $R_{a,b}$, in various cases. In this paper, we improve the 0-estimates of the error terms in the asymptotic formulae for $R_{a,b}(x)$ established by E. Cohen in some cases and further improve them on the assumption of the Riemann hypothesis.

1. Introduction. An integer $n>1$ is called square-full if in the canonical representation of n into prime powers each exponent is $\geqq 2$. Let L denote the set of square-full integers. Let x denote a real variable $\geqq 1$ and let $L(x)$ denote the number of square-full integers $\leqq x$. For the work done on the asymptotic formula for $L(x)$ or for $L_k(x)$, the number of k-full integers $\leqq x$ (an integer $n>1$ is called k-full, if in the canonical representation of n each exponent $\geqq k$) we refer to the bibliography given by E. Cohen [2] and by E. Cohen and K. J. Davis [3]. In particular, for the best known results on the 0-estimates of the error term in the asymptotic formula for $L(x)$, we refer to the paper by the author and R. Sita Rama Chandra Rao [7] and also to the recent paper by the author [8].

In 1963, E. Cohen [1] generalized square-full integers in the following way: Let a and b be fixed positive integers. Let $n=p_1^{a_1}p_2^{a_2}\cdots p_r^{a_r}$ and let $R_{a,b}$ denote the set of all integers n with the property that each exponent a_i $(1\leqq i\leqq r)$ is either a multiple of a or is contained in the progression $at+b$, $t\geqq 0$. It is clear that $R_{2,3}=L$. Let $r_{a,b}$ denote the characteristic function of the set $R_{a,b}$; that is, $r_{a,b}(n)=1$ or 0 according as $n\in R_{a,b}$ or $n\notin R_{a,b}$. Also, let $R_{a,b}(x)$ denote the number of integers $n\leqq x$ such that $n\in R_{a,b}$. The following results have been established by E. Cohen (cf. [1], Theorems 2.1, 3.1 and 3.2):