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GENERALIZED HOMOTOPY EXCISION THEOREMS MODULO A SERRE CLASS OF NILPOTENT GROUPS

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We combine two well known homotopy equivalences of Ganea and some recent work on nilpotent spaces to give a common procedure for deriving the connectivity of generalized excision maps, given that the spaces involved are nilpotent rather than simply connected.

The main results are stated in the context of Serre classes of *nilpotent* groups. Our proof of the Blakers-Massey theorem appears to be new, and applies moreover to any map of nilpotent spaces which induces an epimorphism of fundamental groups (cf. [18, Corollary 6, p. 487]). We close with a very general Freudenthal Suspension Theorem.

M. Mather [16] has proved a theorem on mapping cones wich yields two basic weak homotopy equivalences of Ganea [9, Theorem 1.1] ([10, Theorem 1.1]) as special cases. In the first part of this paper we apply the former equivalence to study the excision maps related to a fibration, generalizing Serre's classical theorem. (The second part uses [10, Theorem 1.1] to derive a generalized Blakers-Massey theorem related to a cofibration.) We have strived to emphasize a parallel treatment in the organization of this paper. Thus each part begins with a discussion of when the *two* generalized excision maps (associated to a fibration (cofibration)) are homomorphisms of *nilpotent* groups, in case $\mathscr{C} \neq \{0\}$: see Corollary 2; and then we show that it is only necessary to determine the mod $\mathscr{C}$ connectivity of one of them: see Proposition 4. Ganea's equivalences are then used to derive the mod $\mathscr{C}$ connectivity of one of the associated maps: see Lemma 5. Finally we induct on the number of stages in a principal refinement of a Postnikov tower of a nilpotent space (the dimension of a finite CW complex) to prove the general result: see Theorems I.12, II.7. (We chose a cellular argument over one using a homology decomposition as it avoids simple connectivity and is less technical.) The problem of finding a common procedure for deriving these excision theorems was raised in [15, p. 52].

We give a brief review of how our work is related to [1], [3], [11], and [17]. Associated to any space B and a fibration $F \xrightarrow{j} Y \xrightarrow{p} X$, there is a transgression square