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THE R-BOREL STRUCTURE ON A CHOQUET SIMPLEX

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The R-Borel structure on a Choquet simplex K is studied. It is shown that the central decomposition and maximal measures coincide, and this is used to improve the well-known theorem that maximal measures are pseudo-concentrated on the extreme boundary.

1. Introduction. Let K denote a compact convex subset of a locally convex Hausdorff topological vector space, and denote by $A^b(K)$ the Banach space of bounded real valued affine functions on K. The symbols $A(K)$, $A(K)^m$, and $A(K)_m$ denote respectively the sets of continuous, lower semi-continuous and upper semi-continuous functions in $A^b(K)$. Set $S(K) = A(K)^m + A(K)_m$, and let $S(K)^\mu$ be the smallest subset of $A^b(K)$ containing $S(K)$ and closed under the formation of pointwise limits of uniformly bounded monotone sequences. $S(K)^\mu$ is a Banach space, the following properties of which were obtained in [6].

THEOREM 1.1. *Consider* $a \in S(K)^\mu$.
(i) $||a|| = ||a \mid \partial_e K||$.
(ii) $a \geqq 0$ *if and only if* $a|\partial_e K \geqq 0$.

$S(K)^\mu$ is an order unit space and thus possesses a *centre* $Z(S(K)^\mu)$ defined in terms of order bounded operators [2]. However a more convenient formulation was obtained in [6]: $z \in S(K)^\mu$ is said to be a *central element* if and only if to each $a \in S(K)^\mu$ there corresponds $b \in S(K)^\mu$ satisfying $b(x) = a(x)z(x)$ for all $x \in \partial_e K$. $Z(S(K)^\mu)$ is then seen to be an algebra and a lattice with operations defined pointwise on $\partial_e K$.

Let π^s be the map which restricts elements of $S(K)^\mu$ to functions on $\partial_e K$. The following representation of $Z(S(K)^\mu)$ as an algebra of measurable functions on $\partial_e K$ was proved in [6]. The statement has been modified slightly to suit the purpose of this note.

THEOREM 1.2. *There exists a σ-algebra $\mathscr{R}$ of subsets of $\partial_e K$ such that π^s is an isometric algebraic isomorphism from $Z(S(K)^\mu)$ onto the algebra $F(\partial_e K, \mathscr{R})$ of bounded $\mathscr{R}$-measurable functions on $\partial_e K$. There exists a unique affine map $x \longrightarrow \nu_x$ from K into the set of probability measures on $\mathscr{R}$ satisfying, for $z \in Z(S(K)^\mu)$,*

$$z(x) = \int_{\partial_e K} \pi^s(z) d\nu_x \, .$$