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PRODUCTS OF BANACH SPACES THAT ARE UNIFORMLY ROTUND IN EVERY DIRECTION

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It is shown that the product of a collection of Banach spaces that are uniformly rotund in every direction (URED) over a URED Banach space need not be URED; this answers a question raised by M. M. Day. A positive result under an additional hypothesis is also proved.

Introduction. A Banach space B is *uniformly rotund in every direction* (URED) if and only if, for every nonzero member z of B and $\varepsilon > 0$, there exists a $\delta > 0$ such that $\|(1/2)(x + y)\| \leqq 1 - \delta$ whenever $\|x\| = \|y\| = 1$, $x - y = \alpha z$ and

$$\|x - y\| \geqq \varepsilon .$$

This generalization of uniform rotundity was introduced by Garkavi [3] to characterize Banach spaces in which every bounded subset has at most one Čebyšev center. Zizler [6] and Day, James, and Swaminathan [2] have investigated this geometrical notion more fully. The purpose of this note is to answer negatively the following question raised by M. M. Day [1, p. 148]: Is the product of a collection of URED Banach spaces over a URED Banach space still URED? In §1, a positive result is proved under an additional hypothesis; the counterexample, §2, is present exactly when this hypothesis fails.

Let S be an index set. A *full function space X on S* is a Banach space of real valued functions f on S such that for each f in X, each function g for which $|g(s)| \leqq |f(s)|$ for each s in S is again in X and $\|g\| \leqq \|f\|$.

Note that X has a natural Banach lattice structure with positive cone $\{f \in X: f(s) \geqq 0 \text{ for all } s \in S\}$ and that X is order complete by its fullness. It follows easily from theorems of Lotz [4, p. 121] and McArthur [5, p. 5] that the following are equivalent:

(1) X contains no closed sublattice order isomorphic to ℓ^∞.

(2) Each order interval in X is compact.

If for each s in S, a Banach space B_s is given, let $P_X B_s$, the *product of the B_s over X*, be the space of all those functions x on S such that (i) $x(s)$ is in B_s for each s in S, and (ii) if f is defined by $f(s) = \|x(s)\|$ for all s in S, then f is in X. For each x in $P_X B_s$, define $\|x\| = \|f\|_X$. With the above definitions, $(P_X B_s; \|\cdot\|)$ is a Banach space.