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COEFFICIENT ESTIMATES FOR CERTAIN MULTIVALENT FUNCTIONS

RONALD J. LEACH

We prove the Goodman conjecture for a class of multivalent functions including close-to-convex functions under the restriction that the coefficients are real. We obtain similar results for other classes of multivalent functions.

1. Introduction. Let S denote the class of all functions f analytic and univalent in the unit disc U with $f(0)=0$ and $f'(0)=1$ and let S^* and K denote the subclasses of starlike and close-to-convex functions, respectively. Several authors ([3], [5], [6], [9]) have defined multivalent analogs of these subclasses. A commonly used definition is that $f \in S(p)$, the class of p-valent starlike functions, if and only if there are numbers z_j with $|z_j| < 1$ and a function $g \in S^*$ such that

$$f(z) = \prod_{m=1}^{p} \psi(z, z_m) g(z)^p \ , \tag{1.1}$$

where

$$\psi(z, z_m) = (z - z_m)(1 - \bar{z}_m z)/z \ .$$

A function $g(z)$ is said to be a Bazilevic function of order α, $\alpha > 0$, if

$$g(z) = \left[\alpha \int_0^z \sigma^\alpha(\xi) h(\xi) \xi^{-1} d\xi \right]^{1/\alpha} ,$$

where σ is a univalent starlike function and $\operatorname{Re} h(z) > 0$, $h(0) = 1$.

If g belongs to the class $B(p)$ of univalent Bazilevic functions of order p, then f belongs to the class $K(p)$ of p-valently close-to-convex functions [6]. The representation (1.1) holds for multivalent analogs of several classes of vnivalent starlike functions [7]. A similar representation holds for the class $V_k(p)$ multivalent functions of bounded boundary rotation, with f and g replaced by their derivatives [8].

We say that a function f belongs to the class $M(p)$ of multivalent functions of order p if there are $z_1, \cdots, z_p$ with $|z_m| \leqq 1$ and a function $g \in S$ such that (1.1) holds. We note that $S(p)$ and $K(p)$ are proper subsets of $M(p)$.

This paper is divided into four sections. In § 2, we prove a preliminary result on the coefficients of a polynomial in $z(1-z)^{-2}$. In § 3, we obtain the Goodman conjecture for functions in $M(p)$ with