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IDEALS AND RADICALS OF SOME ENDOMORPHISM RINGS

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Let R be the full ring of endomorphisms of a reduced abelian p-group G. A description is given, in terms of its action on G, of the Jacobson radical $J(R)$ of R for the case that G is sufficiently projective. Other ideals and radicals of R and their relation to $J(R)$ are discussed.

1. The results. In order to study the structure of a ring R, one of the most important tools is the investigation of radicals. In this context, the following ideals are of interest. Throughout, the word ideal will mean two-sided ideal.

$N(R)$: sum of all nilpotent ideals of R;

$B(R)$: intersection of all prime ideals of R (Baer radical);

$L(R)$: sum of all locally nilpotent ideals of R (Levitzki radical);

$K(R)$: sum of all nil ideals of R (Koethe radical);

$J(R)$: sum of all quasi-regular ideals of R (Jacobson radical).

One has

$$N(R) \subseteqq B(R) \subseteqq L(R) \subseteqq K(R) \subseteqq J(R)\ , \tag{1.1}$$

and all of these ideals, with the possible exception of $J(R)$, are nil [4, pp. 193-197].

In this note we consider the case where $R = \operatorname{End} G$ is the full ring of endomorphisms of a reduced p-primary abelian group G. Our interest will be focused primarily on the Jacobson radical.

In [3] and [9], respectively, lower and upper bounds for $J(\operatorname{End} G)$ where given, namely

$$I(G) \subseteqq J(\operatorname{End} G) \subseteqq H(G) \tag{1.2}$$

which are defined as follows. Note that we write mappings to the right; throughout, λ is a nonzero ordinal such that $p^\lambda G = 0$. Then let $I(G)$ be the set of all $\varepsilon \in \operatorname{End} G$ for which there exists a finite sequence of ordinals $\sigma_0, \sigma_1, \cdots, \sigma_{k+1}$, where $k = k(\varepsilon)$, such that

$$0 = \sigma_0 < \sigma_1 < \cdots < \sigma_k < \sigma_{k+1} = \lambda$$

and, for $i = 0, \cdots, k$, $p^{\sigma_i}G[p]\varepsilon \subseteqq p^{\sigma_{i+1}}G$. Let $H(G)$ be the set of all $\varepsilon \in \operatorname{End} G$ such that, for each nonnegative integer n, $p^n G[p]\varepsilon \subseteqq p^{n+1}G$. Clearly, both $I(G)$ and $H(G)$ are ideals of $\operatorname{End} G$. R. S. Pierce has shown that $J(\operatorname{End} G) = H(G)$ if G is torsion-complete, and $J(\operatorname{End} G) \subsetneqq H(G)$ if G contains an unbounded direct summand which is a direct