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COMMUTATIVE NON-ARCHIMEDEAN C^*-ALGEBRAS

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Commutative non-archimedean C^*-algebras are defined, their properties established, and a representation theory is developed for them. Their closed ideals are completely analyzed in terms of the closed subsets of the spectrum where they 'vanish.' A large class of C^*-algebras is exhibited. A Stone-Weierstrass theorem generalizing a result of Kaplansky is proved.

Introduction. In this paper F denotes a complete non-archimedean valued field, and it is assumed that the valuation is non-trivial. *A non-archimedean normed vector space* over F is a vector space X with a norm satisfying the *strong triangle inequality* $||x + y|| \leqq \max(||x||, ||y||)$ for all $x, y \in X$. If X is complete, X is called a *Banach space* over F.

Let A be an associative algebra over F, and suppose that $||\cdot||$ is a norm on A making A a non-archimedean normed space. If for all $x, y \in A$, $||xy|| \leqq ||x||, ||y||$ (and if A is unital, $||1|| = 1$), then we call A a *non-archimedean algebra*. If, further, A is a Banach space, then we call A a *Banach algebra*. In this paper a Banach algebra will be understood to be commutative and unital unless the contrary is explicitly assumed in a particular context.

If A is a unital commutative C^*-algebra over the complex numbers $\boldsymbol{C}$, then the Gelfand-Naimark theorem shows that if T is the spectrum of A, then A is isometrically isomorphic to $C(T, \boldsymbol{C})$, the algebra of continuous functions on T with values in $\boldsymbol{C}$. In this paper we define a class of algebras, called L-algebras, which play an analogous role in the non-archimedean theory to that played by the algebras $C(T, \boldsymbol{C})$ in the theory over $\boldsymbol{C}$. We prove a Stone-Weierstrass theorem concerning these algebras, and we establish their properties. In the second section we give an abstract definition of a non-archimedean commutative C^*-algebra. Such a definition has been sought for a number of years. We show that every C^*-algebra can be represented by an L-algebra. We derive a number of interesting properties of these C^*-algebras, and in the third section we give some examples of C^*-algebras.

1. The Stone-Weierstrass theorem.

DEFINITION 1.1. A *bundle* is a family $(X_t)_{(t \in T)}$ of Banach algebras