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RADII OF CONVEXITY FOR CERTAIN CLASSES OF UNIVALENT ANALYTIC FUNCTIONS

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Let $P(\alpha, \beta)$ denote the class of functions $p(z)=1+b_1z+\cdots$ which are analytic and satisfy the inequality $|(p(z)-1)/\{2\beta(p(z)-\alpha)-(p(z)-1)\}|<1$ for some α, β $(0\leqq\alpha<1, 0<\beta\leqq1)$ and all $z\in E=\{z: |z|<1\}$. Also, let $P_b(\alpha, \beta)=\{p\in P(\alpha, \beta): p'(0)=2b\beta(1-\alpha), 0\leqq b\leqq 1\}$. In the present paper, we determine sharp estimates for the radii of convexity for functions in the classes $R_a(\alpha, \beta)$ and $S_a^*(\alpha, \beta)$ where $R_a(\alpha, \beta)=\{f(z)=z+a\beta(1-\alpha)z^2+\cdots: f'\in P_a(\alpha, \beta), 0\leqq a\leqq 1\}$, $S_a^*(\alpha, \beta)=\{g(z)=z+2a\beta(1-\alpha)z^2+\cdots: zg'/g\in P_a(\alpha, \beta), 0\leqq a\leqq 1\}$. The results thus obtained not only sharpen and generalize the various known results but also give rise to several new results.

1. Introduction. Let P denote the class of functions

$$p(z)=1+b_1z+b_2z^2+\cdots \tag{1.1}$$

which are analytic and satisfy $\operatorname{Re}(p(z))>0$ for $z\in E\equiv\{z: |z|<1\}$. Considerable work has been done to study the various aspects of the above mentioned class (see e.g., [11], [12] and others). Some of these results have also been extended to the class $P(\alpha)$ of functions $p(z)$ which are analytic and satisfy $\operatorname{Re}(p(z))>\alpha$, $0\leqq\alpha<1$ for $z\in E$. If $p\in P(\alpha)$, it is easily seen that $|b_1|\leqq 2(1-\alpha)$. Further, we note that if $\tau=\exp\{-i \arg b_1\}$ then $p(\tau z)=1+|b_1|z+\cdots$ and so while studying $P(\alpha)$, there is no loss of generality if one takes the first coefficient b_1 in (1.1) to be nonnegative.

McCarty in [8] determined a lower bound on $\operatorname{Re} zp'(z)/p(z)$ for functions $p(z)$ in the class $P_b(\alpha)\equiv\{p\in P(\alpha): p'(0)=2b(1-\alpha), 0\leqq b\leqq 1\}$. He also applied the results obtained to determine the sharp estimates for the radii of convexity of the two classes $R_a(\alpha)$ and $S_a^*(\alpha)$ for each $a\in[0, 1]$ and $\alpha\in[0, 1)$ where

$$R_a(\alpha)=\{f(z)=z+a(1-\alpha)z^2+\cdots: f'\in P_a(\alpha)\}$$

and

$$S_a^*(\alpha)=\{g(z)=z+2a(1-\alpha)z^2+\cdots: zg'/g\in P_a(\alpha)\}\,.$$

For still another class $R_a'(\alpha)$ defined by $R_a'(\alpha)=\{f(z)=z+a(1-\alpha)z^2+\cdots: |f'(z)-1|<\alpha, 1/2<\alpha\leqq 1, z\in E\}$ Goel [4] determined the radius of convexity.

In the present paper, we propose an approach by which it is not only possible to have a unified study of the above mentioned