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A REPRESENTATION OF H^p-FUNCTIONS WITH $0<p<\infty$

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Let E be an open arc in the unit circle. Let F belong to the Hardy space H^p, $0 < p < \infty$, and let g be the restriction of the boundary distribution of F to E. For each $0 < \lambda < 1$ we construct functions $G_\lambda \in H^p$ from g such that $G_\lambda \to F$ in the topology of H^p as $\lambda \to 1$.

I. Introduction. The purpose of this article is to extend to the case $0 < p < 1$ the following theorem of D. J. Patil.

THEOREM A. [2, Th. I, p. 617]. *Let E be a subset of the unit circle T, of positive Lebesgue measure. Let $1 \leqq p \leqq \infty$, let F be in the Hardy space H^p, and let g be the restriction to E of the boundary-value function of F. Denote the normalized Lebesgue measure on T by m, the open unit disc in the complex plane by U, and define for each $\lambda > 0$*

$$H_\lambda(z) = \exp\left\{-\frac{1}{2}\log(1+\lambda)\int_E \frac{w+z}{w-z}\,dm(w)\right\} \qquad (z \in U),$$

$$G_\lambda(z) = \lambda H_\lambda(z)\int_E \frac{\overline{h_\lambda(w)}g(w)}{1-\bar{w}z}\,dm(w)\,, \qquad (z \in U)\,,$$

where h_λ is the boundary-value function of H_λ.

Then as $\lambda \to \infty$, G_λ approaches F uniformly on compact subset of U. Moreover, if $1 < p < \infty$ then $\|G_\lambda - F\|_{H^p} \to 0$ as $\lambda \to \infty$.

The extension of the above to the case $0 < p < 1$ involves a strengthening of the hypotheses: the set E of positive measure will be replaced by an open arc in T, and instead of the characteristic function of E we will work with an infinitely differentiable function with support in E.

Specifically, let E be an open arc in T, and let ψ be an infinitely differentiable function on T with support in E such that

(i) $0 \leqq \psi(w) \leqq 1 \quad (w \in T)$,

(ii) $J = \{w \in T: \psi(w) = 1\}$ has positive Lebesgue measure.

THEOREM B. *Let $0 < p < \infty$, let F be in H^p, and let g be the restriction to E of the boundary distribution of F on T. Define for each $0 < \lambda < 1$*