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STRUCTURE OF Γ-RINGS

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In the first part of the present paper, Γ-rings are studied in the setting of modules. The notion of a module over a Γ-ring is studied, with the object of developing the notion of a Jacobson-radical for a Γ-ring *via* modules. This radical enjoys the usual properties of the corresponding object in rings. A semisimple right Artinian Γ-ring turns out to be the direct sum of simple ideals; this conclusion is strengthened to include a corresponding decomposition for the R-ring Γ also in the case of a strongly semisimple strongly right Artinian weak Γ_N-ring. The Jacobson radical of a weak Γ_N-ring R is characterized in different ways, in one of them as the set of all properly quasi-invertible elements of R. It is shown how rings, ternary rings and associative triple systems can be considered as weak Γ_N-rings. The present approach provides a uniform module *cum* radical theory not only for Γ-rings, but also for the associative triple systems.

The second part of the paper imbeds any weak Γ_N-ring R into a suitable associative ring A. Simplicity and semisimplicity in R and A are shown to be related. The main result of this part which generalizes the classical Wedderburn-Artin theorem for rings to Γ-rings, characterizes the strongly simple, strongly right Artinian weak Γ_N-rings as the Γ-rings of rectangular matrices over division rings.

The ring of all square matrices over a division ring plays a vital role in classical ring theory. However, when one considers the set of all rectangular matrices (of the same type), there appears to be no natural way of introducing a binary ring multiplication into it. Various authors like Nobusawa [15], Lister [8] and Hestenes (see [5]) have tried to offset this difficulty by considering a natural ternary multiplication in the set of rectangular matrices; their investigations have led to the respective notions of a Γ_N-ring, associative triple systems of first kind (ternary rings) and of second kind. These three structures provide a suitable setting for the study of rectangular matrices. The above-mentioned authors have obtained some structural results for these structures, results similar to ones for rings. The concept of weak Γ_N-ring introduced in this paper includes all the three above structures, besides rings, as particular cases. Nobusawa considers a notion of semisimplicity for his Γ_N-ring and that does not arise from a radical as in the case of rings. Coppage and Luh [2] have considered a few radicals among which