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VECTOR FIELDS AND EQUIVARIANT BUNDLES

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We introduce a new method which gives an easy computation of the Chern classes of V-equivariant bundles on the zero set of the holomorphic vector field V. By using this method we obtain the theorem of Riemann-Roch and Hirzebruch for V-equivariant bundles from the holomorphic Lefschetz fixed point formula in case V has arbitrary isolated zeros.

Introduction. For a given holomorphic vector field V on a complex manifold X, a holomorphic vector bundle $E \to X$ is said to be V-equivariant if there exists a $\boldsymbol{C}$-module homomorphism $\hat{V}: \underline{O}_X(E) \to \underline{O}_X(E)$ such that $\hat{V}(fs) = V(f)s + f\hat{V}(s)$ where f(resp. s) is a local section of $X \times \boldsymbol{C}$(resp. E). The importance of V-equivariant bundles comes from the fact that Chern numbers of such bundles are determined on the zeros of V.

In §1, we give cohomological and geometrical obstructions for a holomorphic vector bundle E to be V-equivariant.

In §2, the holomorphic Lefschetz fixed point formula for the arbitrary isolated fixed points is stated. A generalization of a theorem of N. R. O'Brian [6] is given.

Finally we show how the theorem of Riemann-Roch and Hirzebruch can be obtained naturally from the holomorphic Lefschetz fixed point formula via holomorphic vector fields.

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1. Equivariant vector bundles. Let X be a complex manifold, and let V be a holomorphic vector field on X. A holomorphic vector bundle $E \to X$ is said to be V-equivariant, if there exists a $\boldsymbol{C}$-module homomorphism $\hat{V}: \underline{O}_X(E) \to \underline{O}_X(E)$, ($\underline{O}_X(E)$ is the sheaf of holomorphic sections of E), such that $\hat{V}(fs) = V(f)s + f\hat{V}(s)$ where f(resp. s) is a local section of $X \times \boldsymbol{C}$(resp. E). Such a $\hat{V}$ is called a V-derivation on E.

If $\hat{V}$ is a V-derivation on E, then it induces naturally a V-derivation $\hat{V}$ on $\underline{O}_{P(E)}$ where $P(E) \to X$ is the bundle of projective spaces associated to $E \to X$ and $\underline{O}_{P(E)}$ is the sheaf of holomorphic functions on $P(E)$. Conversely it is not hard to see that any V-