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HYPERSPACES OF COMPACT CONVEX SETS

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The purpose of this paper is to develop in detail certain aspects of the space of nonempty compact convex subsets of a subset X (denoted cc(X)) of a metric locally convex T.V.S. It is shown that if X is compact and dim $(X) \geqq 2$ then cc(X) is homeomorphic with the Hilbert cube (denoted cc$(X) \cong I_\infty$). It is shown that if $n \geqq 2$, then cc(R^n) is homeomorphic to I_∞ with a point removed. More specialized results are that if $X \subset R^2$ is such that cc$(X) \cong I_\infty$ then X is a two cell; and that if $X \subset R^3$ is such that cc$(X) \cong I_\infty$ and X is not contained in a hyperplane then X must contain a three cell.

For the most part we will be restricting ourselves to compact spaces X although in the last section of the paper, § 7, we consider some fundamental noncompact spaces.

We will be using the following definitions and notation. For each $n = 1, 2, \cdots, R^n$ will denote Euclidean n-space, $S^{n-1} = \{x \in R^n: \|x\| = 1\}$, $B^n = \{x \in R^n: \|x\| \leqq 1\}$, and ${}^0B^n = \{x \in R^n: \|x\| < 1\}$. A *continuum* is a nonempty, compact, connected metric space. An *n-cell* is a continuum homeomorphic to B^n. The symbol I_∞ denotes the *Hilbert cube*, i.e., $I_\infty = \prod_{i=1}^\infty [-1/2^i, 1/2^i]$. By I_∞^0 we will denote the *pseudo interior* of the Hilbert cube, $I_\infty^0 = \prod_{i=1}^\infty (-1/2^i, 1/2^i)$. We let I^+ denote the set of natural numbers. We use cl and $\overline{\text{co}}$, respectively, to denote closure and closed convex hull. If Y is a subset of a space Z, then int$[Y]$ means the union of all open subsets of Z which are contained in Y. The notation $X \cong Y$ will mean that the space X is homeomorphic to the space Y.

All spaces are considered in this paper to be subsets of a real topological vector space. Since we are restricting our attention in this paper to separable metric spaces this is no restriction topologically or geometrically (cf. Vol. I of [14, p. 242]). If X is a space, by *cc*(X) we will mean the hyperspace of all nonempty compact convex subsets of X (with the Hausdorff metric). We will call *cc*(X) the *cc-hyperspace of* X.

If x and y are points in a real topological vector space V, then $\widehat{xy}$ or $[x, y]$ denotes the *convex segment or point* (if $x = y$) *determined by* x and y, i.e., $\widehat{xy} = \{tx + (1-t)y: 0 \leqq t \leqq 1\} = [x, y]$. Let $X \subset V$. If $x \in X$, we let $S(x)$ denote $\{y \in X: \widehat{xy} \subset X\}$, and we let Ker($X$) denote $\bigcap_{x \in X} S(x)$; the set Ker(X) is called the *kernel of* X. We say X is *starshaped* if and only if Ker$(X) \neq \emptyset$. For $A \subset Y$, a point p in A is called an *extreme point of* A if and only if no convex segment lying in A has p in its (relative) interior. The