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AMENABLE GROUPS FOR WHICH EVERY TOPOLOGICAL LEFT INVARIANT MEAN IS INVARIANT

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Let G be an amenable locally compact group. It is conjectured that every topological left invariant mean on $L_\infty(G)$ is (topologically) invariant if and only if $G \in [FC]^-$. This conjecture is shown to be true when G is discrete and when G is compactly generated.

1. Introduction. Let G be an amenable locally compact group and let $\mathfrak{L}_t(G)(\mathfrak{R}_t(G))$ be the set of topological left (right) invariant means on $L_\infty(G)$. A natural question to ask is: when does $\mathfrak{L}_t(G) = \mathfrak{R}_t(G)$? Obviously, $\mathfrak{L}_t(G) = \mathfrak{R}_t(G)$ if G is compact or abelian. The results of this paper strongly support the conjecture that $\mathfrak{L}_t(G) = \mathfrak{R}_t(G)$ if and only if $G \in [FC]^-$, the class of those locally compact groups each of whose conjugacy classes is relatively compact. Theorem 3.2 (Theorem 4.4) establishes this conjecture when G is discrete (compactly generated).

The present writer's interest in the above question arose from his inability to prove [1, Theorem 7]. The latter result asserts that if G is an exponentially bounded discrete group, then $\mathfrak{L}_t(G) = \mathfrak{R}_t(G)$. This result is false. (See (3.3).)

I am indebted to Dr F. W. Ponting for help in translating portions of [1].

2. Preliminaries. The cardinality of a set A is denoted $|A|$. Let G be a group. The identity of G will be denoted by e, and if $x \in G$, then $C_x = \{yxy^{-1}: y \in G\}$ is the conjugacy class of x in G. If $a, x \in G$, then

$$C(x) = \{y \in G: xy = yx\}\ , \quad C_a(x) = \{y \in G: yxy^{-1} = a\}\ .$$

Now let G be a locally compact group. The family of compact subsets of G is denoted by $\mathscr{C}(G)$ and the family of compact neighborhoods of e in G is denoted by $\mathscr{C}_e(G)$. The algebra of continuous, bounded, complex-valued functions on G is denoted by $C(G)$. Throughout the paper, λ will be a left Haar measure on G. The group G is called an $[FC]^-$ group if C_x is relatively compact for all $x \in G$. The class of discrete $[FC]^-$ groups is denoted by $[FC]$. The group G is called an $[IN]$ group if there exists $D \in \mathscr{C}_e(G)$ such that $xD = Dx$ for all $x \in G$. (For information about the classes $[FC]^-$ and $[IN]$,