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EMBEDDING LATTICES INTO LATTICES OF IDEALS

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A lattice L is transferable iff, whenever L can be embedded in the ideal lattice of a lattice M, then L can be embedded in M. This concept was introduced by the first author in 1965 who also proved in 1966 that in a transferable lattice there are no doubly reducible elements. In fact, he proved that every lattice can be embedded in the ideal lattice of a lattice containing no doubly reducible elements. In a recent paper of the first two authors, the idea emerged that one should study transferability via classes K of lattices with the property that every lattice is embeddable in the ideal lattice of a lattice in K. This approach was used to establish that transferable lattices are semi-distributive. This investigation is carried further in this paper. Our main result shows that every lattice can be embedded in the ideal lattice of a lattice satisfying the two semi-distributive properties and two variants of Whitman's condition.

1. Introduction. It was shown by G. Grätzer ([6], [7]) that every transferable lattice L satisfies the condition

(X) L has no doubly reducible element.

In fact, he proved a stronger result, namely, that every lattice can be embedded in the ideal lattice of a lattice satisfying (X).

In general, if (P) is a lattice-theoretic property which is preserved by sublattices and which satisfies the assertion

$\mathscr{E}(P)$: every lattice can be embedded in the ideal lattice of a lattice satisfying (P),

then (P) is a property of all transferable lattices. In addition to (X), properties of a lattice L for which this assertion is known to hold include

(SF) L is sectionally finite (that is, all principal ideals are finite);

($SD_\wedge$) for $a, b, c \in L$, $a \wedge b = a \wedge c$ implies that $a \wedge b = a \wedge (b \vee c)$;

($SD_\vee$) for $a, b, c \in L$, $a \vee b = a \vee c$ implies that $a \vee b = a \vee (b \wedge c)$.

That $\mathscr{E}(SF)$ holds is a consequence of P. M. Whitman's embedding