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DIFFERENTIAL SYSTEMS WITH IMPULSIVE PERTURBATIONS

S. G. PANDIT

Properties of solutions of measure differential equations are investigated with emphasis on the impulses. A variation of parameters formula, expressing solutions of nonlinear measure differential system in terms of the solution of linear measure differential system and the strength of the impulses, is developed and a result on the asymptotic stability is established.

1. Introduction. Measure differential equations have been investigated by Das and Sharma [3], Leela [4, 5], Raghavendra and Rao [6] and Schmaedeke [7], among others. These equations provide good models for many a physical and biological system. The fact that their solutions are discontinuous renders the conventional methods of ordinary differential equations unapplicable, and thus their study becomes interesting. In [3-6], the equation

$$Dx = F(t, x) + G(t, x)Du \tag{1.1}$$

is studied as an impulsively perturbed system of the ordinary differential equation

$$x' = F(t, x) \qquad \left(' = \frac{d}{dt} \right) .$$

In [7], it is investigated from the view point of optimal control theory, that is, G is assumed to be independent of x. In this paper, we are concerned with the system

$$Dx = f(t, x) + AxDu + g(t, x)Du , \tag{1.2}$$

which is treated as a perturbed system of the linear system

$$Dx = AxDu . \tag{1.3}$$

This gives a more clear picture of the effect of impulses on the behavior of solutions. Deviations from the conventional theory, which are obviously expected, are noted in particular.

Theorem 2.1 indicates the possible abrupt behavior of solutions of (1.3) at the points of discontinuity of u (see also Remark 2.3 and Example 3.1). Theorem 3.1 is a "variation of parameters formula" for the system (1.2). Using Theorem 3.1 and an auxilliary result (Lemma 2.1), we obtain in Theorem 3.2, asymptotic stability