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DIRECT LIMIT GROUPS AND THE KEESLING-MARDESIC SHAPE FIBRATION

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We show that the Keesling-Mardešić shape fibration has an uncountable number of fibers of different shape type. This is done by showing that an uncountable number of nonisomorphic groups can arise as direct limits of direct limit sequences having all groups $\boldsymbol{Z} \oplus \boldsymbol{Z}$ and all bonding homomorphisms given by one of the two matrices $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ or $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$.

A. Introduction. The notion of shape fibration has been developed by Mardešić and Rushing in [4, 5, 6]. In [4] the following question was raised: Let $p: E \rightarrow B$ be a shape fibration and let $x, y \in B$ be points belonging to the same component. Do the fibers $p^{-1}(x)$ and $p^{-1}(y)$ have the same shape? In that paper, this was shown to be the case if x and y belong to the same path component, and in [5, Corollary 1], this was shown to be the case if x and y belong to a subcontinuum of B of trivial shape. In [3], Keesling and Mardešić gave an ingenious example of a shape fibration with connected base space and showed that it has two fibers of different shape. We show that their example, in fact, has an uncountable number of fibers of different shape.

Let an inverse limit sequence of spaces be given where each space is $T^2 = S^1 \times S^1$ and each bonding map is given by one of the two matrices $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$. Here the given matrices each induce a continuous homomorphism on $\boldsymbol{R}^2$ which in turn defines a continuous homomorphism on T^2 via the covering map $e \times e: \boldsymbol{R}^2 \rightarrow T^2$ where $e(t) = e^{2\pi i t}$. Let X be the inverse limit space of such a sequence. Then the discussion in [3, §§2, 3] and [3, Lemma 1] implies that the Keesling-Mardešić fibration has a fiber homeomorphic to X.

Consider the direct limit sequence of groups where each group is $\boldsymbol{Z} \oplus \boldsymbol{Z}$ and each bonding map is given by the transpose of the corresponding matrix in the inverse sequence above. Then by [3, §5], the first Čech cohomology group $\check{H}^1(X, \boldsymbol{Z})$ of X is isomorphic to the direct limit of this sequence. Since Čech cohomology is an invariant of shape type, it suffices to show that there are an uncountable number of isomorphism classes of such direct limit groups.

Thus the groups we are interested in arise as direct limits of sequences