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SUBOBJECTS OF VIRTUAL GROUPS

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Suppose a locally compact group G (always second countable) has a Borel action on an analytic Borel space S so that each element of G transforms a given measure μ into an equivalent measure. If S_0 is the coset space for a closed subgroup H, then there is a natural action of G on S_0 which comes from translations of G on itself and there is such a quasi-invariant measure. Thus it is reasonable to think of such a space (S, μ), for some purposes, as a generalized sort of subgroup, or a virtual subgroup of G. In fact, the set $S \times G$ can be given algebraic and measure-theoretic structure so that many of the procedures used with subgroups can be carried over to this general setting. There is a general notion of virtual group, not necessarily "contained in" a group, which can be derived from this, and it turns out to include equivalence relations with suitable measures as a special case. These virtual groups appear in studying group representations, operator algebras, foliations, etc. Since there is a general setting for virtual groups, it seems desirable to see whether the intuitive idea of an action of a group as representing a subobject fits into this framework in a compatible way. The purpose of this paper is to show that "images" under homomorphisms, "kernels", etc. do fit together properly.

In this introduction we seek to summarize some of the motivation for the theory and give further explanation of the reasons for developing the results presented in the paper. Let G be a locally compact group, and let N be a closed normal subgroup. Let $\hat{N}$ (the *dual* of N) denote the space of equivalence classes of irreducible representations of N, with the Mackey Borel structure [3]. Suppose $\hat{N}$ is analytic, i.e., that N is a type I group [3]. This is the context of the paper of G.W. Mackey [12], in which he studied the problem of finding $\hat{G}$ in such a case. There is a natural action of G on representations of N: If L is a representation and $x \in G$, let $L^x(y) = (xyx^{-1})$ for $y \in N$. This gives a (right) Borel action of G on $\hat{N}$. If U is an irreducible representation of G, $U|N$ is a direct integral relative to an ergodic quasi-invariant measure on $\hat{N}$. Mackey confined his attention to the case in which for every ergodic quasi-invariant measure λ there is a conull orbit (one whose complement has measure 0). In this case we say the action is *essentially transitive* relative to λ. Mackey takes an arbitrary point in that orbit and his constructions are done with the closed subgroup of G