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ON HOLOMORPHIC APPROXIMATION IN WEAKLY PSEUDOCONVEX DOMAINS

F. BEATROUS, JR. AND R. MICHAEL RANGE

A uniform estimate for solutions to the equation $\bar{\partial}u=\alpha$ in a weakly pseudoconvex domain is obtained, provided that the form α vanishes near the set of degeneracy of the Levi form. Under the additional hypothesis that the closure of the domain is holomorphically convex, analogous estimates are obtained for solutions defined in a full neighborhood of the closure. Applications are given to Mergelyan type approximation problems in a weakly pseudoconvex domain D. In particular, it is shown that any function in $A(D)$ can be uniformly approximated by functions in $A(D)$ which extend holomorphically across all strongly pseudoconvex boundary points. When $\bar{D}$ is holomorphically convex, it is shown that the Mergelyan problem can be localized to a small neighborhood of the set on which the Levi form degenerates.

Introduction. A bounded domain D in C^n has the *Mergelyan Property* if continuous functions on $\bar{D}$ which are holomorphic in the interior of $\bar{D}$ can be approximated uniformly on $\bar{D}$ by functions holomorphic in a neighborhood of $\bar{D}$. For $n > 1$, the first non-trivial domains for which the Mergelyan property was verified were the strictly pseudoconvex ones (Henkin [7], Kerzman [9], Lieb [12]). For a more general class of pseudoconvex domains, Range [14] considered approximation by functions which extend holomorphically across strictly pseudoconvex boundary points. Our interest in the problem was rekindled by an example due to Diederich and Fornaess [3] of a smooth pseudoconvex domain D for which approximation by functions holomorphic in a full neighborhood of $\bar{D}$ is impossible. The failure of the Mergelyan property in this example is intimately connected with the absence of a Stein neighborhood basis for $\bar{D}$. Thus a principal objective of this paper is the removal of the hypothesis of existence of Stein neighborhoods which was required in [14].

The key technical tool in [14] is an estimate for solutions to $\bar{\partial}u = \alpha$ when the support of α is disjoint from the non-strictly pseudoconvex boundary. In the absence of a Stein neighborhood base, such estimates were first obtained by Beatrous [1] by using Kohn's global regularity theorem for $\bar{\partial}$ in the construction of a global Ramirez-Grauert-Lieb type kernel which solves the $\bar{\partial}$-problem with the above mentioned support condition. In the present paper,