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A NOTE ON THE KONHAUSER SETS OF BIORTHOGONAL POLYNOMIALS SUGGESTED BY THE LAGUERRE POLYNOMIALS

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Two new classes of bilateral generating functions are given for the Konhauser polynomials $Y_n^\alpha(x;k)$, which are biorthogonal to the Konhauser polynomials $Z_n^\alpha(x;k)$ with respect to the weight function $x^\alpha e^{-x}$ over the interval $(0,\infty)$, $\alpha>-1$ and $k=1,2,3,\cdots$. The bilateral generating functions (1) and (2) below would reduce, when $k=1$, to similar results for the generalized Laguerre polynomials $L_n^{(\alpha)}(x)$. Furthermore, for $k=2$, these formulas yield the corresponding properties of the Preiser polynomials.

It is also shown how the bilateral generating function (2) can be applied to derive a new generating function for the product

$$Y_n^{\alpha-kn}(x;k)Z_n^\beta(y;l)\ ,$$

where $\alpha,\beta>-1$, $k,l=1,2,3,\cdots$, and $n=0,1,2,\cdots$.

1. Introduction. Recently, Joseph D. E. Konhauser discussed two polynomial sets $\{Y_n^\alpha(x;k)\}$ and $\{Z_n^\alpha(x;k)\}$, which are biorthogonal with respect to the weight function $x^\alpha e^{-x}$ over the interval $(0,\infty)$, where $\alpha>-1$ and k is a positive integer. For the polynomials $Y_n^\alpha(x;k)$, the following bilateral generating functions are derived in this paper:

$$\begin{aligned}\sum_{n=0}^\infty Y_{m+n}^\alpha(x;k)\zeta_n(\boldsymbol{y};z)t^n &= (1-t)^{-m-(\alpha+1)/k}\exp\left(x[1-(1-t)^{-1/k}]\right)\\ &\quad\cdot F[x(1-t)^{-1/k};\boldsymbol{y};zt^q/(1-t)^q]\end{aligned} \tag{1}$$

and

$$\begin{aligned}\sum_{n=0}^\infty Y_{m+n}^{\alpha-kn}(x;k)\zeta_n(\boldsymbol{y};z)t^n &= (1+t)^{-1+(\alpha+1)/k}\exp\left(x[1-(1+t)^{1/k}]\right)\\ &\quad\cdot G[x(1+t)^{1/k};\boldsymbol{y};zt^q/(1+t)^q]\ ,\end{aligned} \tag{2}$$

where

$$F[x;\boldsymbol{y};t]=\sum_{n=0}^\infty \lambda_n Y_{m+qn}^\alpha(x;k)\sigma_n(\boldsymbol{y})t^n\ , \tag{3}$$

$$G[x;\boldsymbol{y};t]=\sum_{n=0}^\infty \lambda_n Y_{m+qn}^{\alpha-kqn}(x;k)\sigma_n(\boldsymbol{y})t^n\ , \tag{4}$$