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COHOMOLOGY OF DIAGRAMS AND EQUIVARIANT SINGULAR THEORY

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The purpose of this paper is to define a cohomology theory for diagrams of simplicial sets that specializes to Illman's equivariant singular cohomology for discrete G. We show that such a theory is representable by a suitable Eilenberg-Maclane object. The paper concludes with a comparison of equivariant singular cohomology and equivariant sheaf cohomology.

We adopt the category theory of Maclane as formulated in "Categories for the working mathematician" and use the framework of Quillen's "Homotopical Algebra."

I. **Preliminaries.** We let Δ be the category of finite ordered sets and SS the category of simplicial sets as in [11]. If A is any category cA will denote the category of cosimplicial objects in A, i.e., $cA = \text{Funct}\ (\Delta, A)$.

If J is a small category JS denotes the small complete and cocomplete functor category Funct (J^{op}, SS) and JA the category of abelian group objects in JS. Furthermore, if $F \in JS$ and $K \in SS$ we define $F \otimes K$ and F^K pointwise by $F \otimes K(j) = F(j) \times K$ and $F^K(j) = F(j)^K$.

JS may be enriched in SS by the functor Nat: $JS^{op} \times JS \to SS$ defined by $\text{Nat}(E, F)_n = \text{Nat}(E \otimes \Delta[n], F)$ where $\Delta[n]$ is the standard n-simplex in SS. Thus JS is a simplicial category in the sense of [14], Chapter II. We note that JS is tensored over SS via $(\ ) \otimes K$ and cotensored over SS via $(\ )^K$.

A strict homotopy is a morphism of the form $F \otimes \Delta[1] \to E$ and gives rise to the strict homotopy relation on morphisms of JS. We let the homotopy relation on morphisms of JS be the equivalence relation generated by the strict homotopy relation. We denote the homotopy category of JS by hJS with Hom sets $h\text{Nat}(E, F)$ abbreviated $h(E, F)$.

A morphism $f: E \to F$ is called a fibration, respectively weak equivalence, if $f(j)$ is a fibration, respectively weak equivalence, for each $j \in J$. A cofibration is a morphism that has the left lifting property with respect to all trivial fibrations. We have the following result of Quillen-Bousfield-Kan [1], pg. 313:

THEOREM 1.1. *JS equipped as above is a closed simplicial model category.*