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A MULTIPLE SERIES TRANSFORMATION OF THE VERY WELL POISED ${}_{2k+4}\Psi_{2k+4}$

STEPHEN C. MILNE

A multiple series generalization of the q-analog of Whipple's theorem is derived for ${}_{2k+4}\Psi_{2k+4}$ by applying recent analytical techniques of Askey and Ismail to Andrews' multiple series transformation of a well poised ${}_{2k+4}\Phi_{2k+3}$.

1. Introduction. The bilateral basic hypergeometric function is

$$ {}_m\Psi_n\left[\begin{matrix} a_1, a_2, \cdots, a_m; q, t \\ b_1, b_2, \cdots, b_n \end{matrix}\right] = \sum_{j=-\infty}^{\infty} \frac{(a_1)_j(a_2)_j \cdots (a_m)_j t^j}{(b_1)_j(b_2)_j \cdots (b_n)_j} , $$

where

$$ (a; q)_\infty = \prod_{n=0}^{\infty} (1 - aq^n) , \qquad |q| < 1 \tag{1.1} $$

and

$$ (a)_j = (a; q)_j = (a)_\infty (aq^j)_\infty^{-1} , \tag{1.2} $$

or

$$ (a)_{-n} = \left(1 - \frac{a}{q^n}\right)^{-1} \cdots \left(1 - \frac{a}{q}\right)^{-1} = (-a)^{-n} q^{n(n+1)/2} (q/a)_n^{-1} . \tag{1.3} $$

Thus

$$ \begin{aligned} &{}_m\Psi_n\left[\begin{matrix} a_1, \cdots, a_m; q, t \\ b_1, \cdots, b_n \end{matrix}\right] \\ &= \sum_{j=0}^{\infty} \frac{(a_1)_j \cdots (a_m)_j t^j}{(b_1)_j \cdots (b_n)_j} + \sum_{j=1}^{\infty} \frac{\left(\frac{q}{b_1}\right)_j \cdots \left(\frac{q}{b_n}\right)_j}{\left(\frac{q}{a_1}\right)_j \cdots \left(\frac{q}{a_m}\right)_j} \left(\frac{b_1 \cdots b_n}{a_1 \cdots a_m t}\right)^j (-1)^{j(m-n)} \\ &\quad \times q^{j(j+1)(m-n)/2} . \end{aligned} \tag{1.4} $$

Hence we see that to insure convergence we must require $n \leqq m$. Also $b_h \neq q^{-N}$, $a_h \neq q^{N+1}$ for any nonnegative integer N. Finally if $n < m$, we need only in addition require $|t| < 1$; however, if $n = m$, we need also

$$ \left|\frac{b_1 \cdots b_n}{a_1 \cdots a_m}\right| < |t| < 1 . $$