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ASYMPTOTIC PROPERTIES OF NONOSCILLATORY SOLUTIONS OF HIGHER ORDER DIFFERENTIAL EQUATIONS

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A classification of the nonoscillatory solutions based on their asymptotic properties of the differential equation $y^{(n)} + py = 0$ is discussed. In particular, the number of solutions belonging to the Kiguradze class A_j is determined.

We investigate asymptotic properties of the nonoscillatory solutions of the differential equation

$$y^{(n)} + py = 0 \,, \tag{E}$$

where p is a continuous function of one sign on an interval $[a, \infty)$. Various aspects of Eq. (E) have been investigated by a number of authors [1-15]; in most cases, under the condition that the integral

$$I(r) \equiv \int_a^\infty x^r |p(x)| dx \tag{1}$$

is either finite or infinite for some constant r. For instance, Eq. (E) is oscillatory on $[a, \infty)$ if the integral (1) is infinite with $r = n - 1 - \varepsilon$ for some $\varepsilon > 0$ [4, 8]. On the other hand, if $I(n - 1)$ is finite, (E) is nonoscillatory; in fact, it is eventually disconjugate [9, 14, 15]. Under the same condition, results on the existence of a fundamental system of solutions possessing certain asymptotic properties have also been obtained [5, 13]. Of particular interest to the present work, however, is the notion of class A_p introduced by Kiguradze [4] with the help of inequalities in Lemma 1.

A solution of (E) is said to be *nonoscillatory* on $[a, \infty)$ if it does not have an infinite number of zeros on $[a, \infty)$. (Unless the contrary is stated, the word "solution" is used as an abbreviation for "nontrivial solution.") Eq. (E) is said to be nonoscillatory on $[a, \infty)$ if every solution of (E) is nonoscillatory on $[a, \infty)$. If there exists a point $b \geqq a$ such that no solution of (E) has more than $n - 1$ zeros on $[b, \infty)$, Eq. (E) is said to be *eventually disconjugate* on $[a, \infty)$.

As previous studies of Eq. (E) indicate, asymptotic properties of the solutions strongly depend on the parity of n and the sign of p. For this reason, it is convenient to classify Eq. (E) into the following four distinct classes:

$$n \text{ even}, \qquad p \geqq 0 \,, \tag{i}$$