PACIFIC JOURNAL OF MATHEMATICS
Vol. 93, No. 2, 1981

DISTINGUISHING A PLANE CURVE FROM OTHER CURVES SIMILAR TO IT

M. S. KEANE AND S. J. SIDNEY

If C is a convex or smooth simple closed plane curve, then there is a finite subset E of C such that any plane curve which is similar to C and passes through all the points of E must coincide with C. Generalizations to compacta in euclidean spaces are given; on the other hand, there are simple closed plane curves for which no such finite subsets exist. Circles are the only plane continua which separate the plane for which three points suffice, and even for a convex polygon the number of points required may be arbitrarily large.

1. **Introduction.** A circle is determined by (any) three of its points. The aim of this paper is to examine analogous statements for other compact sets in a euclidean space, and especially for more general simple closed plane curves. Precisely, if K is compact, must K contain a finite subset E such that whenever K' is similar to K and contains all the points of E, it follows that $K' = K$? In general the reply is negative even for simple closed plane curves (Example 7), but the following result (the original objective of this work) is true.

THEOREM 1. *If C is a convex or smooth simple closed plane curve, then C contains a finite subset E such that $C' \sim C$ and $E \subset C'$ together imply $C' = C$.*

Here and throughout, $\sim$ denotes similarity and "smooth" means continuously differentiable. In §2 we prove generalizations and refinements of Theorem 1 in stages, using finite subsets of C to successively force C' to be congruent to C, have the same "spanning circle" as C, and finally coincide with C. It will become clear that, for general K, the only possible obstacle to an affirmative reply to our question is the absence of any finite subset (consisting of at least two points) of K of which K contains a smallest copy (Theorem 5). Taking advantage of this perspective, in §3 a result (Theorem 6) is proved which shows that, for instance, rather general piecewise smooth objects can be added to the list begun in Theorem 1, and a simple closed plane curve for which no finite subset works is constructed. Section 4 is devoted to estimating how large a finite subset is required for various particular plane sets; in particular (Theorem 8) if a continuum separates the plane, it requires at least four points