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CLOSED ULTRAFILTERS AND REALCOMPACTNESS

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We introduce some conditions which are closely related to closed ultrafilters and establish interconnections among these conditions and characterize realcompactness, alomost realcompactness, c-realcompactness and weak-cb^*-ness, $\cdots$.

Throughout this paper, by a space we mean a completely regular Hausdorff space and all functions are continuous and we assume familiarity with [3] whose notation and terminology will be used throughout. For a given space X, we denote by βX (or υX) the Stone-Čech compactification (or realcompactification) of X. In §1, we give definitions and preliminaries and introduce some conditions which are closely related to closed ultrafilters. In §2, we establish interconnections among conditions introduced in §1. In §3, we characterize realcompactness, almost realcompactness c-realcompactness and weak-cb^*-ness and give some examples in §4.

Notations and terminologies. $N =$ the set of positive integers, nbd $=$ neighborhood, $\omega =$ the first countable ordinal, $\Omega =$ the first uncountable ordinal, $C(X) =$ the ring of all continuous functions on X, $Z(f) =$ the zero set of $f \in C(X)$ where we assume $0 \leqq f \leqq 1$, $Z(X) =$ the set of all zero sets, $X^* = \beta X - X$. $\mathscr{F}$($\mathscr{U}$ or $\mathscr{R}$ resp.) $=$ a free closed (open or regular closed resp.) ultrafilter. $\mathscr{F}^p(\mathscr{Z}^p) =$ a free closed (Z) ultrafilter converging to $p \in X^*$. $\mathfrak{F} =$ the set of all $\mathscr{F}$ (similarly define $\mathscr{U}^p$, $\mathfrak{U}$ and $\mathfrak{R}$ resp.), $\mathrm{cl}\mathscr{U} = \{\mathrm{cl}\, U;\ U \in \mathscr{U}\}$ and $\{F_n\}_{\mathrm{cl}} \downarrow$ ($\{F_n\}_{\mathrm{cl}} \downarrow \varnothing$) $=$ a decreasing sequence of closed sets (with the empty intersection). Similarly we define $\{R_n\}_{rc} \downarrow$ and $\{Z_n\}_{ze} \downarrow \cdots$ where "rc" and "ze" denote "R_n is a regular closed set" and "Z_n is a zero set" respectively.

1. Definitions and preliminaries. A family $\mathscr{A}$ of subsets of X is said to be *stable* if for any $f \in C(X)$ there is $A \in \mathscr{A}$ such that $f \mid A$ is bounded. Mandelker ([10], Th. 5.1) has proved that X is realcompact iff any stable closed family $\mathscr{A}$ with the finite intersection property has non-empty intersection and Hardy and Woods ([4], Lemma 2.6) have obtained that $\mathscr{R}$ is stable iff there is $p \in \upsilon X - X$ and $\mathscr{R} \to p$. We say that $\mathscr{U}$ or $\mathscr{R}$ has *CIP* if $\cap\, \mathrm{cl}\, A_n \neq \varnothing$ for any